Deep Mathematics and Emergent Meta-Systems Theory

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Introduction

In this paper we will explore deep mathematics and connect that with the theory of meta-systems. Deep mathematics is a new extension of the domain of mathematics through the consideration of ontological matters that are foundational for Systems Theory. We extend the Systems Theory or George Klir in Architecture of Systems Problem Solving with a Meta-systems Theory. We then place both Systems Theory and Meta-Systems extensions in a broader mathematical framework. Most of this deepening of mathematics follows from the question concerning what is more fundamental than mathematical Category Theory. We attempt to develop this foundation in the context of the development of a theory of Emergent Meta-Systems. Emergent Meta-Systems stem from the recognition of three special systems that act as a hinge between Systems and Meta-systems. The attempt to understand this hinge drives us to understand the possibility of non-duality and attempt to frame a mathematics that can approach the comprehension of the non-dual.

Metaphysics and Mathematics

First we define the metaphysical in the terms used by Anaxamander as the distinction between the Apeiron (Unlimited) and Peiron (Limited). There are many metaphysical principles in Western Philosophical history, but the Apeiron was the first of these. The most common interpretation of the metaphysical principle is Being. The peiron is distinguished again between the Logos (upwelling of speech) and Physus (growth of physical things). Once these two dualities are defined there are myriad interpretations of the different terms involved and their relations to derivative terms. However, we will take the position that all these interpretations form a field. The field is such that each distinction like a mobius strip is locally dual and globally non-dual. But around the mobius strip every possible demarcation is at one time or another designated as real. In order to avoid this situation we posit that it is possible to prevent the logos from interacting with the physus thus preventing dualism from taking hold. In such a case we recognize nomos as the non-dual order that is archaic prior to the logos/physus distinction. Similarly we can disarm the dualism between the Unlimited and the Limited by forcing them apart and thus recognize the fundamental Indo-European non-dual moment prior to the arising of the Apeiron/Peiron distinction called RTA. RTA means cosmic harmony in the Indo-European tradition and stands for the source of order. RTA is translated in our modern English language as Right which designates the preferable asymmetry and that which is contextually correct based on principle.

Given this metaphysical landscape we can immediately recognize the nomos as the object of Mathematics as a discipline. Nomos is ordering which lies behind both the unfolding of the Logos and the Physus. It is because the Nomos is prior that physical states of affairs can be connected by mathematics to our theories that unfold in the logos of scientific discourse. If the nomos were not prior to the arising of both aspects of our duality at the epistemic level this connection would not be possible. The non-dual RTA is the principle that lies behind that non-dual order as its source. RTA operates in the chiasm between the Unlimited and the Limited while the nomos operates in the chiasm between Logos and Physus. Chiasm was a term introduced by Merleau-Ponty to describe a state of Being very close to non-duality. Apeiron/Peiron and Logos/Physus form a mobile. RTA and the Nomos arise within the interspace of that duality when it is disabled. As long as it is operating dualistically these non-dual moments are invisible but as soon as we put the duality out of play then the non-dual moments appear.

As mentioned previously Being is the normal interpretation of the Apeiron. In our time Being has become fragmented. What was once the single highest Concept has now become internally differentiated in a series of steps. The original concept of Being is called by Heidegger in Being and Time a Pure Presence Being (present-at-hand) which he distinguishes from a Process Being (ready-to-hand) as two modalities of Being proper. Being proper is a monolith which embodies transcendence grounding itself. These two modalities operate within the monolith of Being as a kind of higher dimensionality within Heidegger's ontology that solves the problems of Kantian metaphysics without giving up its critical boundaries. Just as a fourth dimensional space allows us to exit a sphere without going through its boundary so the Process Being modality allow us to exit the realm of critical transcendental philosophy without violating its precepts that keep Reason engaged with understanding. Heidegger's monolithic Being with its two modalities appears as an archaic origin prior to the arising of the Subject/Object (logos/physus) dichotomy. In this way it attempts to approach the non-dual by positing that the Apeiron is the Monolith of Being with its dual modes and that the Peiron arises from that giving us the subject/object split which transcendental Critical Philosophy depends upon. Heidegger discovered a ground which combined the views of the Presocratics Parmenides (Pure Presence) and Heraclitus (Process Being) in a neat formulation. Heidegger used this to propose a solution to the great split that had appeared in physics in his day which was between Quantum Mechanics and Relativity Theory. Both posit a globally incoherent non-experience-able state of affairs contrary to the coherent state of affairs what we perceive. In Quantum Mechanics that is the probability wave which is simultaneously in all possible states that collapses to one state randomly when perceived. In Special Relativity Theory that non-experience-able realm is four dimensional spacetime. We experience this background matrix of all phenomena differently from different inertial frames of reference. In this case the Lorenz transformations between different observers is not relative at all but mathematically precise. Thus Special

Relativity Theory is inherently deterministic while Quantum Mechanics is inherently probabilistic. But these two incompatible views of nature are duals of each other, and can be seen as sharing the positing of a non-experiential realm like the monolith of Temporal Being that includes the two modalities of Pure Presence and Process Being. Our subject/object experiences arise out of this more archaic level of Being. Our experiences are objectively three dimensional slices of a four dimensional spacetime matrix. Our experiences are subjectively observations that collapse the probability waves that inhabit the four dimensional realm. In either case it is our experience that brings the phenomena out of the Monolith of modal Being into one of its modes. One of those modes is like the nouns of language while the other is like the verbs. In other words we either have an emphasis on operands or operations. But the Monolithic Being itself is unified in the sense that it is both an operator and an operand at the same time. This, in fact, is the Indo-European ideal of wholeness. This fact that Heidegger's philosophy was designed to solve the quandary of the duality between quantum mechanics and special relativity is a little known facet of his philosophy. Heidegger, studied physics after Theology and before Philosophy and had hoped to become a Physics Professor. Philosophy was his third choice. His philosophically sophisticated and subtle solution says that Special Relativity Theory and Quantum Mechanics are duals of each other like the particle/wave or the measurement of position or velocity. They both arise from the same source. One is probabilistic because it is rooted in Process Being and the other is deterministic because it is rooted in Pure Presence. Being itself is split into two modalities in a complementary fashion. The two major views of physical theory that extend the Newtonian paradigm arise from the duality inherent in the monolith of Being.

All would be well if there were only two modalities of Being within the Monolith. But continental philosophy discovered that the Monolith breaks up by the emergence of two more levels of even more radical difference that that suggested by the term mode. Rather quickly it was recognized that Process Being had an antinomy called Nothingness by Sartre in Being and Nothingness. Although, Heidegger attempted to dismiss Sartre's less sophisticated brand of Existentialism, it became clear that the antinomy was very significant and in fact broke the Monolith of Being into pieces. Thus, with the advent of what Merleau-Ponty called the Hyper Dialectic between Process Being and Nothingness in The Visible and the Invisible, and what Heidegger called Being (crossed out) a new kind of Being appeared and converted the previous modalities into kinds as well. Derrida capitalized on this work and defined what he called differAnce as the supplement of differing and deferring which he explored in terms of the fundamental difference in the Western tradition between the place of Speech verses the place of Writing. I call this kind of Being -- Hyper Being and assign it the modality 'In-hand' because it is the point where radical transmutation takes place and the technology that appears in the ready-to-hand of Process Being transform in our hands. The being-in-the-world, i.e. Dasein (being there)

which is the human prior to the arising of subject/object duality has different modalities. For Pure Presence the modality of being-in-the-world is called present-at-hand by Heidegger which Merleau-Ponty explains psychologically in Phenomenology of Perception as "pointing." For Process being the modality of being-in-the-world is called ready-to-hand by Heidegger and interpreted psychologically by Merleau-Ponty as "grasping." I augment this by giving Hyper Being the modality of being-in-the-world as the 'in-hand' and giving the psychological interpretation taken from Levinas as 'bearing'. Levinas says that in this third kind of Being, Hyper Being, is the point where Metaphysics and Ethics collapse together.

But the radical difference of kinds that arises between the modes of Being at the third meta-level of Being is not the end of the emergent series. At the fourth meta-level a new even more radical difference I call the Integra arises. The integra is an ordering that goes deeper than essences and orders the individual at fractal levels within the accidents that make up the individual. This produces a new integral unfolding of Being which Merleau-Ponty calls Wild Being in The Visible and the Invisible. I give this final sort of Being a modality of being-in-the-world called "out-of-hand" and say that its psychological interpretation is "encompassing". Wild Being is almost impossible to think and all the higher meta-levels of Being are actually impossible to think, similar to the unthinkability of Bateson's fifth meta-levels of learning and motion in physics as described in Steps To the Ecology of the Mind. Wild Being is almost non-dual while the fifth meta-level of Being actually goes beyond Being into the realm of existence and is actually non-dual. We interpret this existence of the fifth meta-level of Being as over the edge and outside Being proper in terms of the Buddhist concept of sunyata (emptiness). This makes our ontology empirical in the sense that anyone can try to 'think' the fifth meta-level of Being and thus expand our world. But as long as this remains a challenge unmet then we say that what is at the fifth meta-level and above is outside Being. Being is not 'n' dimensional but only has four meta-level emergent steps. After that the stairs stop abruptly and whatever is beyond that is outside our worldview. We give the name Existence to what lies beyond the pale of the highest threshold of Being.

Being proper has four mode/kind/integra differentiated moments. It also has four aspects related to reality, identity, truth, and presence. We define existence as that which is neither true nor false, real nor unreal, identical nor different, present nor absent. Being proper is the process of presentation and ideation. That is Being proper produces illusions. What goes beyond Being and has only existence is what we find (Arabic wujud) without presentation (Arabic kun) mixed in. When the Arabs interpreted the Aristotle they noticed that Being did not exist in their language. Thus they invented the technical term Kun (made thing) to stand for Being in order to differentiate it from Wujud (existence) found in Arabic naturally. Then, when this scholarship was taken back into the Latin the word Existence was coined to stand for Wujud in contradistinction to the essences that had Being. There has been an implicit tension between essence and existence ever since which came to a

head when Jaspers made an issue of it. Then the existentialists followed emphasizing existence over essence as Sartre did. Heidegger accommodated the two and interpreted existence as the production of subjectivity/objectivity out of the self-grounding monolith of Being. Thus Heidegger interpreted existence in terms of ex-stasis or ecstasy of the projection of reified beings by our being-in-the-world out of the Modal Monolith, the It of It Gives or what Heidegger latter called Appropriation in contra distinction to the enframing of the experiential Newtonian realm.

Pure Presence	Process Being	Hyper Being	Wild Being
No difference	Modal difference	Kinds or essential difference	Integral difference
Present-at-hand	Ready-to-hand	In-hand	Out-of-hand
Point	Grasp	Bear	Encompass
Determinate	Probability	Possibility	Propensity
Form	Pattern	Trace	Disposition/ Tendency

Now that we have defined the integral kinds of Being and shown how the undifferentiated and continuous concept of Being that existed up to the present century has become fragmented in modern Continental ontology we can frame our major mathematical point. All mathematics is basically Present-at-hand. Mathematics has not made the fundamental transformation that both Philosophy and Physics have made into comprehending phenomena thorough different ontological modalities. Every mathematical formalism is setup as purely present and operated solely within that modality as if that were the only ontological modality that existed. This historical fact makes Mathematics with all its differentiation merely a surface phenomena considering the unexplored depths that lay beyond the formal surface. Beneath the surface of form there lies three other levels in the Nomos that are unexplored hither to in the Western Mathematical tradition. It is the purpose of this paper to lay out these other ontological levels of possible mathematical elaboration as we move from pure duality and pure continuity toward the non-dual and chaotic discontinuity. Here we have identified a new frontier. We will survey that new landscape but it is so vast that it will have to be left to others to discover what other wonders exist there.

Unfolding the Nomos

We have identified that all mathematical categories hither to identified and explored exist only in one ontological layer. We will trace a route to the deepening of Mathematics starting with category theory. That route will take us away from our idealized Eldorado into a jungle that is ignored by mathematical formalism and logic. Or at most mathematics constructs images of these other levels within the realm of pure presence. Mathematics never leaves its haunts where in formalisms can be easily built. These other ontological levels are successively harder to think about less well build mathematical representations in so as to explore the nomos. As we move through our exposition of the unfolding of the nomos we will contrast that with the unfolding of the physus and the logos. In the realm of the physus we have formal structural systems theory which we apply as a methodology and in the realm of the logos we have logic. To set the stage we will describe the segmentation of our worldview.

In the Western worldview there is as we have said the realm in which we make distinctions. These distinctions are designated as real, or identical, or true,

or present. Against this backdrop there is the continual pressure of analysis against the physus by the continual use of reductive methods. Theories on the other hand have to withstand the continual pressure of skepticism. We apply this reduction in the following manner. We first identify a form and attempt to construct a formalism within which proofs are possible to understand it. If we cannot understand the form through a formalism then we reduce that form to its contents and attempt to use a category scheme of contents to explain it away as a means of understanding. If we cannot reduce it structurally then we fall back on description of the wholes that the form is part of using systems theory as an underpinning. Formal Proof is the strongest strategy of understanding, Structural Explanation is the second strongest strategy, and finally Systemic Description is the weakest strategy for attempted understanding. Basically if we cannot prove we fragment in order to explain and if we cannot explain then we accept wholeness. This fall back to wholes as a last resort causes us to posit emergent ontic levels in the phenomena like:

Ontic (physus)	**Ontological (logos)**
(Gaia)	Pluriverse
Social	Kosmos
Organism	World
Multi-Cell	Domain
Cell	Meta-System
Macro-Molecule	System
Molecule	Form
Atom	Pattern
Fundamental Particle	Monad
Quark	Facet
(Sub-quark)	

There is no inherent relation between the two sides of this dualism. We get them mixed up all the time saying that Systems and Forms exist in nature. But fundamentally the Ontological are templates for understanding that may be applied to any level of the ontic hierarchy. The ontic hierarchy is produced by the failure to be able to analyze and reduce everything. What ever withstands extreme attempts at reduction is generally accepted as emergent ontic levels in the physus. What keeps the Emergent Ontological levels independent of each other is a healthy skepticism that every theory that unfolds from the logos must face. The ontological emergent levels are merely common strategies of understanding at various levels of complexity. All these levels are advanced tentatively in order to give us something to talk about but are not in any sense final or demonstrated to have any kind of identity, truth, reality or presence in themselves. They are merely designated true, present, real, or identical for the purposes at hand.

The strategies for understanding and a healthy skepticism concerning their application is what gives us systems theory and its extension up and down the ontological layers. These appear as models in the theoretical logos. The inner coherence of these is maintained in terms of Logic. We contrast this to what becomes present phenomenologically which is tested to ascertain its reality on the side of physus. The move from what appears as the thresholds of complexity of theory toward logic on the one hand is the dual of the move from what is presented through testing toward the reality of the ontic emergent levels that inform the physus. So the hierarchy of models is contrast to the logic that gives it coherence on the one hand. The hierarchy of emergent phenomenal levels is contrast by reductive testing to what appears. Between these two realms

of physus and logos appears the non-dual nomos that informs each. We get at the nomos by using mathematics as a means of rigorously underpinning our models. We use experimentation to verify that these models actually capture what is happening in reality behind the appearances. Ultimately the Kantian will tell us that the transcendental object is a noumena that cannot be apprehended either by reason or experience. Also the Kantian will tell us that the transcendental subject is similarly inaccessible. It is the transcendental God that maintains the coherence between these two syntheses in the world behind the scenes. But Heidegger tells us that exactly that thing which Kant took out of his metaphysics, the transcendental imagination, posits a prior and what J.G. Ballard calls archaic state before subject and object arise to give us the duality and the unsolvable problem. All we have to do is return to the monolith in which subject and object duality does not exist and our metaphysical problem with transcendence never becoming immanence is solved without sacrificing the necessary combination of reason and experience that is the foundation of understanding. So as we look at the unfolding of the nomos exposed by deep mathematics we will track that against the models and the logic that stands on the side of logos and compare that to the phenomenology and unbracketing or designation as real, identical, true, absent that takes us out of the phenomenological mode.

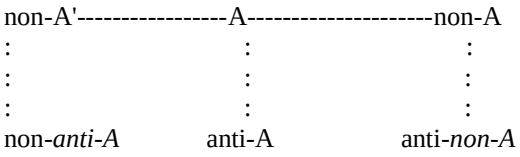
As we look at the unfolding of the nomos we will start with category theory. We know that category theory is the current foundation of mathematics and we know that it allows the commutative property to slip but holds onto the associative property. Thus if we want to produce a move toward the indeterminate away from the determinate present-at-hand basis of current mathematics then we will have to give up the assumption of the associative property. This will produce something similar to a figure found in logic called the Square of Contraries and Contradictions that is used by Greimas for a structural analysis of narrative. We follow Greimas and note that this structural analysis based on a form from logic is precisely what we need to move away from formalism to the level of pattern. Grenander in his books on Pattern Synthesis is the only mathematician we know to attempt to produce a mathematics of patterning. He does this by producing a series of pattern generators, similar to Klir's generative models that produce data patterns. The generators exist at the present-at-hand level but the patterns themselves exist at the structural level. In fact we will differentiate four different kinds of patterning called, sign, value, process and structure. It is these kinds of patterning that produce the transformations in content that underlie form. These transformations become apparent when we emphasize the dualities between forms and their opposites and between a pair of form/anti-form and everything else.

The Greimas Square has the following form:

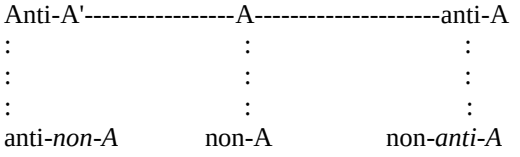
A---------- non-A
: :
: :
: :
: :
anti-A-------- anti-*non- A*

In the Greimas square the contradictory of ‘anti’ is ‘non’ and the contradictory of

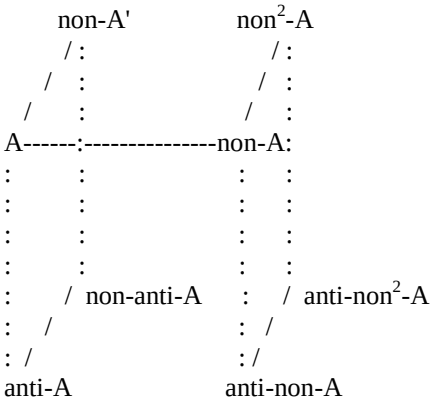
A is anti-non-A which is the Other. A and anti-A are contraries as are non-A and non-anti-A. We unfold the Greimas square if we differentiate anti-non-A from non-anti-A. This unfolding is called chiasmic and represents the reversibility at the level of Wild Being that Merleau-Ponty calls 'Flesh' in The Visible and the Invisible. The unfolded Merleau-Ponty/Greimas 'book' appears thus:



There is a dual of this book which has the form:



These two books may be combined to produced the Unfolded Greimas Cube which has the form:



This cubic formation has the form of the Octonion. It combines the two Greimas book duals into a single self interfering formation that is chiasmic in all its unfolded directions. It is like a glass cube with warpages in the glass. Each direction of unfolding holds a chiasmic reversibility and these phase transitions within their respective intervals interfere with each other at the center of the cube giving the embodiment of Wild Being as a complex and chaotic phase space.

What we want to do is recognize a similar unfolding of category theory in which we posit that association no longer holds. This is precisely the property loss that generates the octonion from the quaternion. We want to apply this property loss to standard Mathematical Category Theory and see what happens. In other words arrows will no longer be able to be reversed to produce the dual of the category. With this loss much of the explicit power of Category Theory is lost so it is no wonder this kind of transformation was not considered before. We must remember that we have both arrows within categories and arrows between categories. We will demand that both kinds of arrows lose their reversibility. This is similar to the anti-book and the non-book duals of the Greimas square unfolding. We suggest the name Proto-Category Theory for the discipline which loses association in both arrows and functors. Note that this loss is at the global level not at the local level. So there are islands of association similar to the island where the commutative property holds despite a global failure. The failure of a property globally causes a kind of global incoherence which may be saved locally. The relation between local coherence and global coherence is the dual of the local non-duality verses

the global non-duality of the mobius strip. Thus the Penrose triangle (Escher waterfall) is the dual of the mobius strip in some sense. It is the ability to maintain the property locally in spite of global failure that produces all the wondrous variety of categories and we expect it to produce a corresponding variety of proto-categories.

We use the term proto-category as opposed to category in order to harken back to the difference between the gestalt and what I call the proto-gestalt, which is a temporal gestalt. The temporal gestalt is what organizes the showing and hiding of gestalts within manifestation. It has what David Bohm called an implicate order. In other words there is a hidden order that underlies the specific gestalts and organizes our movement from one gestalt to another. In the case of mathematics these gestalts are the different categories, like set, group, lattice, etc. There is a field of these categories and the categories themselves are organized within this field on an unknown basis of ordering. Why do we have these specific categories with this specific ordering. That is because the categories that exist are unfoldings of the underlying nomos in different ways. The fact that there are functors between categories shows us that the same underlying patterning mechanism is appearing in multiple categories in slightly different but recognizably the same incarnations. When we move into the field theory underlying the categories what we hope to do is get closer to the source of all these similar projections. However that lies beyond the disintegration of category theory into proto-category theory. We must move beyond the gestalts of the categories with their specific embodiments of order to the seeming disorder of the non-associative field of proto-categories in order to understand the source of the ordering of the field that sustains the categories and organizes our passage between the different categories. Different categories are useful for different things. Our thoughts move between the categories looking for templates that will aid our understanding. This movement between categories encounters the proto-gestalt which appears in the interstices between the categories. There is a netherworld of proto-categories that lack the associative property that is completely hidden by present-at-hand mathematics. Present-at-hand mathematics only considers the islands of the categories not the ocean of ambiguity between the categories that we encounter every time we leave the safe harbor of a specific category and set sail for another category in our search for an understanding of all the facets of mathematics or in search of prototypes of order by which to understand some phenomena that we are studying. In that sea there are myriad shadows of the categories which haunt the pristine designated as real categories themselves in which fuzziness and ambiguity reigns instead of the clear and distinct forms of order that the categories embody.

First we lose the associative property so that our arrows do not reverse, so that the dualities of our categories break apart, then the arrows of the different morphisms become probabilistic and then fuzzy and finally chaotic. When we lost the commutative property we allowed for a much greater range of categories to enter mathematics. Now when we lose the associative property and enter the realm of proto-categories an even greater number of possible

categories become visible. But eventually the very morphism lines themselves begin to move through the process of unraveling as they become probabilistic entering into the realm of Process Being, then they become fuzzy entering into the realm of Hyper Being, and finally they become Chaotic entering into the realm of Wild Being. Past that chaotic stage of the unraveling of the morphisms of our proto-category theory we say that the morphisms themselves become empty because the lines of relation that make up our category theory become completely unstable and undependable. At that point we reach the limit of our ability to model the nomos.

How are we to understand this limit that we have reached and the stages that we have gone through to reach it? First we note that we have step by step unfolded the Greimas Square into the Unfolded Greimas Cube. In other words we know that Category theory is made up of dual categories produced by reversing their arrows. These dual categories like set and anti-set are seen in terms of the Greimas square as A and ~A. The pre-set of the categories are seen as Non-A. But when we lose the associative property we find that there is a difference between Anti-Non-A and Non-Anti-A. In other words when we look at the gestalt of a single category its anti-category and the other possible categories form the background of our delimited exploration of a single category. We know we can get to the anti-category by reversing the arrows. We know we can get to the other categories by following functors. But when the arrows lose their associative property we can no longer count on these morphisms and we fall from the level of form to the level of pattern in our consideration of the ordering from the nomos. The emergent ontological template of understanding through forms encompasses forms in space and forms in time. Forms in time are actions. When we loose the commutative property we allow forms in time to have different effects when reversed. When we loose the associative property however forms in space also disintegrate because we cannot cross a boundary and re-cross it to get back to the same place. The is the whole concept behind Spencer-Brown's Laws of Form where he delineates that crossing and re-crossing a distinction in null. When we lose the associative property the context on the outside of the form effects the crossing and re-crossing. We fall into the dual of the Laws of Form which I call the laws of pattern. In the laws of pattern layering is affirmed while multiplicity is denied, while in the laws of form multiplicity is affirmed while layering is denied. The two sets of laws are as follows:

Laws of Form
()() = ()
(()) = null

Laws of Pattern
()() = null
(()) = ()

When we lose the Commutative property in the production of Category Theory we are allowed to create a kind of bridge between all the different manifestations of nomos. The points of reference vanish in favor of a formalism based on morphisms. The actions within a formalism are allowed to be non-commutative. This relaxation of the basic operation of the formalism admits many new formalisms into a designated as real status within mathematics and

also allows us to understand the relations between all the defined formalisms. But relaxing the associative property takes us out of the emergent level of form all together. It relaxes the associative property. Suddenly our bridges between categories in the form of functors are no loner reversible. Our dualities between category and anti-categories breaks down as well. Suddenly the context between categories is important and effects a meta-morphism transition between categories. Suddenly the dual can break into multiple images of the other as it is infected by different categories that it intermingles with in the background of the gestalt of the within which it is a figure. All this brings us down to the emergent cognitive level of pattern from the level of form. Since we can not cross out of the form and back again with impunity and since the context outside the category suddenly matters we are thrown into the opposite of the laws of form where the nesting of categories suddenly matters as in the laws of pattern. In this state our categories gain depth as does the field between them. Different routes though the landscape of categories yield different results and paths across the landscape cannot be reversed easily. We begin to see categories within categories within categories as the nested categories reflect the whole field of categories. Suddenly the field transforms into an interpenetrating jeweled network in which every embodiment of order reflects the rest of the embodiments of order. All the functors become internalized as reflections of other categories within the field. Since we cannot traverse the field with impunity any longer we switch to seeing the mirrorings of the field within the individual category which is deepened to allow for other categorical aspects within a single category. The duals by becoming not immediately reversible appear as a cloud of different alternatives that encompass aspects from other categories. So eventually we see that the non-anti-A is different from the anti-non-A as the variety of the categories beyond the one in question are taken into the relation between the specified category and its dual. This opens up a chiasmic field within each category in which the field of categories enters into the category itself producing a set of reversibilities with identifiable phases in which pure associative reversibility has been lost so that the phase of association between categorical nodes is differential from the opposite or returning phase of association. The full logic of this internal separation of the Category from its opposite via non-associative arrows and its reflection in the field of all the categories to which it is intrinsically related via non-associative functors is embodied in the Unfolded Greimas Cube. That cube has multifarious reversibilities which all interfere with each other to produce the image of Wild Being. Pure Presence Being is the level at which there is a clear delineation between a category and its opposite and with the field of all the other categories on which it is defined. Process Being appears in the movement of the morphisms itself either within a category or between categories. Hyper Being appears in the non-decidability between a category and its anti-category, or between two categories within the field of possible categories. Wild Being appears suddenly when we drop the associative law that links a category to its opposite or between categories within the seascape of island categories. Wild Being appears in the chiasmic

reversibility that is not associative between a category and its dual and the nesting of other categories within that dual OR between a category and another category in which all the other categories interfere. This move toward Wild Being in mathematics is recapitulated in the indeterminacy of the arrows themselves. In fact what is occurring is that not only do the categories undergo the opening up of chiasmic reversibility but the arrows undergo a similar process except in their case it is expressed in the arrows losing their purely present determinism. The arrows enter into Process Being when they become probabilistic. They enter into Hyper Being when they become fuzzy. They enter into Wild Being when they become Chaotic that is when they exemplify a mixture of order and disorder.

Now we might ask why is it that the opening up of proto-category theory has this specific kind of structure rather than some other. Why must we lose the commutative property in order to establish the categorical imperative which links all the possible categories and gives us a universal mathematical language for comprehending them. But then what happens when we go on to lose the associative property in our transition to the underlying field that supports the categories. Finally the arrows lose their definiteness and disintegrate though a series of similar steps taking us though probabilities, fuzzification and finally into chaos. The reason behind this has to do with the way we construct our world. Our Indo-European world is constructed in layers and when we explore the basis of category theory we are opening up the deeper layers of Being within our world. We peal back these layers one at a time until we get to the point where all the difference is lost between the categories and we enter a pure fusion of all the categories with all the other categories and our ability to distinguish between them and divide them from one another is completely lost. You would think that below the level of pure presence which normal category theory enjoys there would be nothing but vague chaos and indeterminacy. But in fact there is gradual shading off of order and that fading of order is part of the nomos itself as well. In fact we find an important aspect of the nomos here which is its non-duality. So that is why it is important to us to explore the field within which the categories arise and the transitions between categories as well as just the present-at-hand categories themselves. It gives us some understanding of the deeper levels of the nomos beyond the category theory and it gives us some comprehension of the underlying structure of our world.

Now in order to understand why the deep structure underlying mathematics has the form that it does we must delve into the distinction already breached between a System and a Meta-system. We have breached it when we talked about a category as being a gestalt and we contrasted that to the proto-gestalt which we enter when we lose the associative property. When we do that we enter the field between the categories themselves and that field itself becomes mirrored and reflected within the individual categories. In other words, the functors become internalized within the category and the dual category becomes differentiated by those reflections so that like a hologram the field of categories is introjected into each category within the field. Proto-category theory becomes a

proto-gestalt in the sense of a temporal gestalt. In other words our path of thought between the categories becomes important suddenly and by taking different paths between the categories we end up in different places. Going out to another category and coming back places us in a different context rather than exactly where we started. Looking at the reflections of other categories in a specific category in different orders changes our comprehension of that category. At the associative level the dynamics of our exploring the landscape of the nomos as it is represented in the field of categories becomes extremely significant. We know that different mathematicians have different view points. What we have not considered is the fact that this is because they have explored the landscape of the nomos in different orders and the viewpoint they have constructed is an artifact of this journey that they have undertaken in their studies. Viewpoints are produced as articulations of the field of the proto-gestalt. The social cohort of living and interacting mathematicians exemplifies the proto-gestalt in the concretization of their different mathematical viewpoints. They know in practice that there are different ways of relating categories and relating to categories and they carefully attempt to hide that in their publications which focus on present at hand results, stripped of the process of discovery and the meanings that attend to their comprehension of the nomos. Each mathematician has to travel through the landscape of the categories and those journeys are all different depending on the paths taken in each specific case. It is not just because mathematicians are different that there is such variety of mathematical viewpoints created. It is because they traverse the landscape in different paths and situate their specialties with respect to different prioritizations of the categories themselves. The mathematical journey directly confronts the proto-gestalt. What we see when we come to set theory from lattice theory is completely different from what we see when we come at it from group theory. All the various categories can be seen though the mediation of the others and that mediation changes our comprehension of the categories which is our specialty.

The purely present mathematical results that appear in articles and books that delineate pristine and pure formalisms hides the actual process of manifestation by which mathematicians undertake their studies and approach their results. In that process of thought they wander though the proto-categorical landscape and attempt to keep in mind the purified and pristine forms of the categories and distinguish them from that ocean of confused resemblances. When we find discontinuities within that landscape where resemblances fail and new categories have to be created then we have encountered Hyper Being. When we realize that the discontinuities and the continuities are fundamentally mixed up in our representations and that the underlying nomos that produces the differences and similarities is non-dual then we enter the realm of Wild Being. When we loose our ability to distinguish even on the level of pattern after we have lost our hold on form completely then we enter the realm of utter non-duality and emptiness where the nomos becomes unthinkable completely.

All this exploration of the underlying basis of mathematics that seeks to look into its ungrounded foundations is

dependent on a simple model that underlies the articulation of our Indo-European worldview. It is expressed in the following figure:

SYSTEM
Pure Presence Being
DISSIPATIVE SPECIAL SYSTEM complexnion
Process Being
AUTOPOIETIC SPECIAL SYSTEM quaternion
Hyper Being
REFLEXIVE SPECIAL SYSTEM octonion
Wild Being
META-SYSTEM

When we speak of the gestalt and proto-gestalt as a way of understanding the difference between associative category theory and non-associative proto-category theory, then we are making in a different way the distinction between a 'system' and a 'meta-system', or what Bataille calls a 'restricted economy' and a 'general' or 'global economy'. The meta-system is the environment, context, or situation that surrounds and encompasses a system. I define a system as a social gestalt from the point of view of social phenomenology. A meta-system is the social situation or social context that surrounds the seeing of a socially designated as real, true, identical or present gestalt from a particular socially constructed viewpoint. The meta-system can be seen either as the side-effects or shadows of the system as it haunts other systems within the categorical field, or as the fragmentation of viewpoints that cast those shadows. Each category is a restricted economy conforming to certain mathematical definitions. But each category participates in an underlying categorical landscape that forms a general economy. The functors are a way of talking about this underlying similarity that comes from the non-dual nomos expressing itself in various categorical embodiments. But when the morphisms break down and become both indeterminate and become non-associative then we enter fully into the meta-systemic proto-gestalt that the proto-category theory manifests. This is the field out of which mathematicians attempt to wrest the discrete and clear categories that they hold in pure presence and display in their papers and books. But each paper and book is a narrative that winds its way though the field of mathematical forms in a certain order. The ordering of the journey effects deeply the results that appear along the way. Ultimately we can see the categorical landscape to be very much like a jungle of possible orderings and each mathematical paper or book cuts a specific path through that jungle seeing intimations of underlying order everywhere but differently than would have been seen on some other path.

What is important to understand is that between the system and meta-system, or between the gestalt and proto-gestalt there are several specific steps that are defined as the special systems and called dissipative, reflexive and autopoietic. Where a system is a gestalt that is a whole greater than the sum of its parts, i.e. having emergent properties, a meta-system is a proto-gestalt with implicate order that is a whole less that the sum of its parts , i.e. having de-emergent lacks. The special systems are defined in terms of algebras so that the dissipative special system is ordered by the algebra of complex numbers, the autopoietic special system is ordered by the algebra of quaternions, and the reflexive special system is ordered by the algebra of octonions. These algebras are dependent on properties and lose properties as we go down the cascade from the system to the meta-system. The dissipative special

system with its complexnion algebra has all the properties of the real numbers that define the system gestalt. The autopoietic special system with its quaternion algebra loses the commutative property. The reflexive special system loses the associative property. The meta-system is associated with all the other non-division algebras that lose the property of division such as the sedenion which is generated by the Cayley Dickson process as the next possible on beyond the octonion. But the infinity of non-associative algebras associated with the meta-system are of little interest to us because they do not allow the intertranformability of numbers by algebraic manipulation that we find so useful and that attract us to these three unique division algebras. The point is that in the transition between the system gestalt and the meta-systemic proto-gestalt we encounter these specific algebras as we lose algebraic properties one by one. The difference between these steps define the kinds of Being. So as we are traversing the series of steps we are also moving from one kind of Being to another until we get to the meta-system that represents pure existence outside of any hint of Being. That existence is unthinkable and can be interpreted as the Buddhists do as sunyata, i.e. emptiness that is itself empty. The kinds of Being define the differences between the special systems as the special systems define the differences between the kinds of Being in a very deep reciprocity. It is this reciprocity that defines the underlying structure below category theory which is poised at the level of pure presence in some ways an process being in other ways.

When we look at category theory we see that it allows the commutative property to relax so that it takes us to the level of quaternions in order to build a bridge of functors between categories and develop a universal language to describe all the mathematical categories. It defines the morphism as something determinate and purely present. The elements vanish in category theory the morphisms become arrows that represent transformational processes in order to build a bridge between the forms of order represented by the categories. But the arrows that represent morphisms are reified and remain present-at-hand. If we seek to deepen category theory that establishes itself on the structural process level we need to release the associative property to become relaxed as well and then release the division property. At the same time we need to allow the arrows to become indeterminate moving though a progression from probabilistic, to fuzzy and finally to chaotic before losing our ability to differentiate them. This progression down though the layers of the kinds of Being of the world and their associated special systems merely completes the journey that category theory itself begun. Category theory is a structural theory which attempts to explain the differences between categories and bridge those differences by constructing a language of morphisms that can be universally applied across all categories. At the same time it relaxes the commutative property in order to allow in all the non-commutative categories. Now we insist that it is necessary to relax the associative property to allow in all the other non-associative proto-categories. When we do that we take ourselves down into the level of Being called Hyper Being. At this level of being there

is indeterminacy between various positions. This indeterminacy opens out its indecisiveness into an articulated field that appears only when we cannot reverse arrows at will. Duals are no longer interchangeable. The dual is invaded by the background of the other categories. Functors become paths of thought between categories that cannot be easily reversed and allow the interembedding of categories. After releasing the associative property we find ourselves at a point where we must release the division property as well so that it becomes impossible to distinguish categories any longer as their mutual mirroring overwhelms our capacity to differentiate. This occurs at the same point that the arrows after becoming probabilistic, fuzzy and chaotic themselves become indistinguishable. At that point we reach the limit of our capacity to differentiate the nomos. On the way to that liminal mathematical state we unearth the non-dual nature of the nomos underneath our projection of the categories as a structural way of comprehending forms of order. It is that non-dual form of ordering that underlies all the dualistic projections that we want to explore more deeply. Realizing that the Unfolded Greimas Cube of reversible chiasmic relations exist within the categorical landscape is a key point in the evolution of our mathematical intuition. Where category theory is poised at the quaternion level algebraically the Unfolded Greimas Cube of non-dualities is located at the octonionic level which is reflexive and inherently social. This is why we develop a social phenomenology to attack that level of the articulation of the proto-gestalt. We define the limits of mathematical thought in the disintegration of the purely present category theory not in order to destroy mathematics but in order to open up a deeper mathematics that is concerned with the non-dual nomos directly embodied rather than merely represented by the reflections of the categories in each other. We will use a series of mathematical analogies in order to explore this non-dual dimension more deeply.

Between the logos and the physus appears the non-dual nomos. Logos is articulated as truth and identity expresses logic as the move from truth to identity. Physus is articulated as reality and presence and expresses the testing of scientific experiment as it moves between the two. Reality is the filtering and testing of the system which is presented to the meta-system. Truth is the relation of the system to the meta-system which attempts to remain true to itself in order to avoid meta-systemic filtering. So when we distinguish between the system and the meta-system we are articulating the spit between logos and physus and the split between the aspects of reality they encompass. All this exemplifies the limited (peiron) as distinct from the metaphysical principle of the transcendent and unlimited which is Being itself as a whole (Apeiron). Between the limited and the unlimited is another non-dual moment called the ***RTA*** (ASA) which we know today as the RIGHT but which used to have the meaning of ‘cosmic harmony’. There is a difference between knowing what should be done, i.e. the nomos, and what is right in any situation. This difference appears outside of mathematics. Plato talks about it in terms of the divided line as the difference between mathematically distinct intelligibilities and

intelligibilities that cannot be described mathematically. When we deepen our mathematics we move beyond the nomos to the level of the source of the Beautiful and on to the source of the Good. The source of the Beautiful is the same as the RTA or right. In the Republic Plato deals with rights in terms of justice. Justice is seen as the distribution of rights among individuals within the city. The best distribution of rights is the most beautiful. In terms of the Parable of the cave there are the objects held by the Sophists which are seen by the bound spectators as appearances. The objects and the appearances are the lower echelons of the divided line. But the upper echelons can be seen in terms of the Fire that allows the objects to be seen and the Sunlight beyond the Cave. We already know from the analogy of the Sun that the Good is thought of as the Sun. So we can think of the source of the Beautiful that we are told to contemplate in the Symposium as the Fire within the Cave. It is the non-dual source of Right. What is right is in harmony with the cosmos. What exemplifies harmony is the most beautiful. Thus rightness has an inner relation to beauty. The source of rightness and beauty are the same. It is symbolized by the fire within the cave that allows us to see the appearances of the objects held by the Sophists. This is why in mathematics we aim for simplicity, beauty and elegance because that is what feels right to our mathematical intuitions. It is in that intuition that the Arte (related to RTA) or excellence of Mathematics inheres. But beyond the Arte by which we intuit the beauty and rightness of mathematics there is the source of the Good. That source is beyond the Cave of Being and its projection machinery that is differentiated by the different kinds of Being. Beyond the cave is existence which is empty and thus interpenetrates. The Good is the source of variety in existence and thus of the variety of the categories. The Good is a cornucopia of variety production which we see at work in the industry of mathematical definition of ever new kinds of categories and elaborations of existing categories. When we move deeper into the Nomos we encounter its limit when it reaches the Beautiful (outwardly) or Right (inwardly). Beyond that is yet another deeper non-duality which is called the Good which is the source from which all the variety comes whether beautiful or ugly or as Socrates says about love in the Symposium "neither". We see ***RTA*** in our will to power seeking elegance, simplicity, beauty and aesthetic appeal in our mathematical and physical theories or formalisms. The non-duality of order looks across to a deeper non-duality of Beauty and aesthetic elegance which we find fitting and which provides our appreciation of mathematical order with some degree of ecstasy. Beyond that non-duality we wonder at the variety of mathematical forms. If God did create with mathematics as his foundation then we find ourselves in awe not just of the variety of created forms but at the variety of the sorts of nomi that exist. We can only discover these deeper non-dualities if we go down into the uncertain realm of the proto-categories. We must enter the self-interfering non-dual chiasms of the Unfolded Greimas Cube in order to find the traces of these deeper non-dual sources that give our mathematical skills and intuitions their reason for the continual exploration of the landscape of the categories. It is the reflections of a deeper order that we see

expressed in the categories that keeps us searching the landscape of mathematics and sends us out on the oceans of thought that separate the categories form each other. What we need is a way to approach that deeper mathematics that unearths the non-dualities more directly.

Non-dual Mathematical Anomalies

From the great field of category theory and its opening up into Proto-category theory as the environment of mathematical thinking we are going to focus now on a few mathematical anomalies that give us intimations of the underlying depths that we have described in general terms. These anomalies give us a view of the non-dual underpinnings of the nomos within specific mathematical contexts which in themselves are dualistic and present-at-hand. We will be focusing on the anomalous construction of the mobius strip within geometry and extending that to higher dimensional geometrical constructions. In general the mobius strip is a precise icon of non-duality because it is locally dual and globally non-dual due to the fact it as only one side and one edge despite its appearance of being two sided and two edged when looked at narrowly. In the mobius strip the one edge wraps around itself to become both edges of the strip and similarly the single side wraps around it to become both sides simultaneously. This occurs by the strips twisting in space and is in fact composed of a single strip of paper twisted once with its ends joined. Mobius strips can be twisted either left or right and when two oppositely twisted strips are joined together then form what is called a Kleinian bottle. The bottle is also a non-dual surface with the added feature of enclosing space and that the same side that encloses space is that which forms the outside of the bottle. We normally hear more about the Mobius Strip than the Kleinan bottle. But Steve Rosen has done a philosophical study of the both the Mobius Strip and the Bottle which shows that it has deep implications for out thinking and especially that it is a good analogy for non-dual thinking. Here we will elaborate on his results. One of the most interesting things he says is that the Bottle is somehow between the third and fourth dimensions. It is this concept that we will explore more deeply in our exegesis.

Dissipative Special Systems

We will start by noting that the dual of the Mobius Strip is the Penrose Triangle. The Penrose triangle is that impossible three dimensional looking drawing of a triangular configuration of slats that appears to turn inside out as we follow it around becoming an impossible figure. Escher used this triangular configuration due to Penrose effectively in his Waterfall Etching. Generally a Penrose triangle is an impossible paradoxical looking figure which makes sense when you look at each vertex but does not make sense when it is looked at globally. Such a figure cannot really exist in three dimensions. The difference between the Mobius strip and the Penrose triangle is that the Mobius strip achieves non-duality in reality while the Penrose triangle is only an illusion of a figure that wraps around itself but which cannot exist in three dimensional space. The Penrose triangle has the property of global incoherence with local coherence. Each vertex appears normal but the global configuration is skewed

impossibly. The difference between local coherence and global incoherence is the opposite of local duality and global non-duality. The non-dual is coherent but in such a way that duality is embedded within it. Local coherence has to do with the configuration of the aspects of the pictured object. When the object is looked at as a whole it lacks coherence even though the individual parts seem to fit together correctly. What the Mobius Strip tells us is that we can have non-duality without losing coherence globally. Also what the Penrose triangle tells us is that losing global coherence comes from the inability of the parts to fit into the whole properly. The Penrose triangle is a figure that does not have holonic properties, in other words the local configuration does not fit into the global configuration. On the other hand the Mobius strip has a peculiar integrity that allows for the two sides to fit together so well as to be one side and the two edges fit together so well as to be one single edge. Integrity and Holonic characteristics are very high on the scale of harmony that Cheng produced where mutual support is the highest level prior to interpenetration. Cheng's levels are strife, logical relation, mutual interaction, mutual support and interpenetration as we move up toward higher and higher levels of harmony. Mobius Strips exemplifies mutual supporting integrity of a peculiar non-dual sort. The Penrose triangle lacks holonic mutual support of parts fitting into the whole.

The Mobius Strip and the Penrose triangle relate to the dissipative special system. That special system has been described by Prigogine in Order Out of Chaos. He calls dissipative special systems 'dissipative structures'. This means that in anomalous far from equilibrium chaotic systems it is possible for order to be produced. That order dissipates outward from the center toward the periphery. Such systems are neg-entropic in a local area. Globally entropy is maintained but in rare conditions local order generation is possible. The order appears from a singularity and move out toward the boundary of the dissipative special system. At the boundary the dissipative special system disorders the environment and reorders it according to the new ordering regime flowing from the negentropic center. Because the boundary is always bigger than the singularity that means the total entropy is always greater than the order created so the general law of entropy is maintained. A good model of the Dissipative Special System in nature is the Soliton wave. Such a wave maintains its energy much longer than one would normally expect. Because of this we consider the soliton super efficient. Such a wave is like a dissipative structure. The boundary is the trough within which it travels. The wave is reflecting off the walls of its trough and continually reconstituting itself from those reflections. The anomaly of the super-efficiency of the soliton wave is off set by the energy necessary to create the tough that it must live within. But in the soliton we see the continual circulation of reflected energy from the soliton to the boundary tough and back to the center as it moves down the trough. As it moves it will go right though another soliton without losing energy, it will turn corners with the channel that it is following and bounce off walls also without losing energy. Soliton solutions exist for many physical equations so their possibility is rife in nature.

Similarly we can say that the Mobius strip is an analogy for the dissipative special system as well which can be modeled mathematically by complex numbers. Complex numbers have an odd twist out of the plane of the real numbers when we take the square root of negative one. The Mobius strip models the twisting around at the boundary and the bouncing back of energy to the singularity where the order is composed and seemingly flows in from nowhere. But another interpretation is that there is a potential well between the boundary and the singularity where the disorder of the boundary twists back into the singularity to become the order that emanates from the singularity. The Mobius strip shows us how the negentropic ordering can be the flip side of disordering so that the same process that disorders the environment can be reflected in the ordering within the dissipative special system.

Although dissipative special systems are negentropic they are not perpetual motion machines. They do not create energy nor do they balance energy. What they do is consume energy but in such a way that spontaneously produces order in a determined small region of the world which feeds on the disordering of larger regions of the world. The perpetual motion machine almost always has a form similar to the illusory form of the Penrose triangle. In the perpetual motion machine an impossible flow is set up to try to recirculate energy impossibly back on itself somehow. We know that there is no perpetual motion machines just as there are no three dimensional Penrose triangles. Both are a trick that is really impossible to pull off. But just because it is impossible to pull off that trick with energy and matter it does not mean that it is impossible with information. Information flow based on consumption of energy may cause negentropy or some other super-efficiency when it takes the form of the Mobius strip becoming like a soliton with the underlying mathematical structure of the complexnions. Dissipative structures are perpetual information flow machines. They take advantage of the fact that chaotic systems can have infinite information as in strange attractors and use energy to promulgate the information flow in ways that appear and actually are super-efficient. Dissipative Special Systems are the systems theoretic prototypes for all such super-efficient perpetual information machines. Although perpetual motion machines are impossible perpetual information conversion machines are not. This difference between possibility and impossibility is reflected in the difference between the Penrose triangle and the Mobius strip. If the waterfall of Escher contained information instead of water or energy then it would be accurate. The point is that even though each individual conversion of the perpetual motion machine might make sense the global incoherence of the machine makes no sense because it violates entropy laws. However, information is disembodied in a way such that for information these violations do not occur. Information transfer uses energy and expends it but the information itself can be conserved or produced by algorithms super abundantly so that information seems to come from nowhere. The difference between the illusion of the Penrose triangle and the Mobius strip is instructive because it is through non-duality that it is possible to produce this

effect of super abundant ordering within a small portion of space based on energy consumption. The perpetual motion machine fails to achieve that nonduality and thus loses global coherence which the Mobius strip achieves. One is a carefully constructed illusion and the other is a realized reality.

Autopoietic Special Systems

Once we understand the relation between the Mobius strip and the Penrose triangle and their relation to the dissipative system that can be said to be analogous to the soliton and complexnions we are ready for the next step. Each of these figures may be related to a further development. Two Mobius strips produce a Kleinian bottle. Similarly there is another paradoxical figure called the Necker Cube which has similar but slightly different properties than those of the Penrose triangle. What is interesting is that when we go up to this new level the properties of the two types of figures are swapped. The Necker Cube is globally dual and locally non-dual while the Kleinian Bottle is globally coherent and locally incoherent. Notice this switching back and forth between the properties of the two dual figures. The Kleinian bottle is globally a coherent figure but locally it has its self-intersection where it is incoherent. With respect to the Necker cube we find that globally there are two ways of looking at the cube that oscillate in our perception. But locally this dual oscillation vanishes when we look at a single vertex. Steve Rosen says that the Kleinian bottle is non-dually between the third and fourth dimension because we could make the self-interference disappear if we moved the neck out of the third dimension either toward 'anna' or 'kappa' into the fourth dimension. However, he says that this makes the actual neck of the Kleinian Bottle disappear and turns it into something else. That something else turns out to be the pentahedron in four dimensional space, i.e. the analog of the tetrahedron which is the simplest polytope in that space being composed of five orthogonal points, ten lines, ten triangular sides and five tetrahedrons combining into one four dimensional figure. The sides of the pentahedron have two sets that can be reduced to two intertwined Mobius strips. The pentahedron has no anomaly of self-intersection. In four dimensional space the pentahedron is a balanced figure with no asymmetries unlike its three dimensional analog the Kleinian bottle. I believe that Steve Rosen has had a deep insight when he says that the Kleinian bottle is non-dually between dimensions. It is both the Kleinian bottle and the pentahedron and its self intersection point is key to understanding non-duality as it manifests in our world. The point of self-intersection is like the concept of a set being a member of itself. Onar Aam pointed out that the set and the Kleinian bottle have similar characteristics and I am extending that analysis with respect to the Hyper-set or the Non-Well-founded Sets of Aczel. Such anomalous sets can be members of themselves or self intersect. When we look at the global duality of the paradoxical figure of the Necker cube and compare it with the Kleinian bottle and its embodiment of local incoherence in space, we are lead to consider Riemann space that is globally incoherent but locally coherent which is more like the Penrose Triangle. Riemann space is the opposite of the Kleinian bottle. When we look at the bottle we see that it is a single surface embedded in a cast of space. Since the

bottle is asymmetrical and has two different ends there are four possible bottles and thus four possible casts of space around such bottles. We can consider the space within which the Kleinian bottle appears to have global curvature like the Riemann space and thus have a fourth invisible dimension. The global curvature is invisible to us but on the other hand the ambiguous space of the self-intersection is very visible. The Kleinian bottle is a single surface but it articulates a three space. Thus it is two dimensional and three dimensional at the same time in different aspects. Riemann space is four dimensional and three dimensional at the same time in its coherent and incoherent aspects. The Necker cube is a three dimensional illusion but actually must inhabit only two dimensional space of representation. The Necker cube is an illusion while the Kleinian bottle is a realizable reality.

This brings us to the consideration of the difference between paradox and super-rationality. Paradox is a sickness of the intellect which creates an illusion of sophistication. Super-Rationality is non-dual and produces a reality that goes beyond dualistic constructs. The Western tradition is fascinated with paradox, as we see in Hofsteader's Godel Escher Bach: Eternal Golden Braid for instance, but has little experience or understanding of super-rationality. Super-rationality was developed to a refined level sophistication in Buddhism. We see the super-rational at work in Zen Koans. In the super-rational we hold two opposite positions at the same time without conflict. In paradoxicality there is conflict between the opposites. The Penrose triangle and the Necker cube are examples of paradoxicality while the Mobius strip and the Kleinian bottle are examples of super-rational non-duality. Notice that the former is always an illusion while the latter is always a realizable anomalous reality. One of the key distinctions on the road to wisdom is the difference between the super-rational and the paradoxical. The latter represents intellectual sickness while the former represents embodied health at a deep level. Our figures tell us how closely intertwined the two enfoldings of reason can be. One takes us into illusion while the other takes us into a reality that is highly integral. The super-rational combines the holonic and the integral into the interpenetrating. The paradoxical takes us into the strife of opposites and mysterious conjunction that has no reality.

The Kleinian Bottle and the Necker cube also relate to the next higher special system called the autopoietic. The autopoietic is a conjunction of two dissipative systems. It abides by the mathematics of the quaternion and is exemplified physically by superconductivity. It is more than super-efficient it is in fact ultra-efficient and violates local entropy completely. Autopoietic systems were defined by Maturana and Varela. They are systems that produce themselves and hold their organization as a homeostatic variable in spite of structural changes below the threshold of their organization. Maturana and Varela's theory is really about autopoietic forms and that is defined by an appeal to the structural level of patterning below form and the organizational or systemic level of holistic ordering above the level of form. Spencer Brown describes these autopoieitc forms very well in Laws of Form. They are forms in which the

operator and the operand are the same. Such a form Spencer-Brown calls a "mark" in his philosophical boundary algebra. Maturana and Varela's theory magically introduces observers at the system level and so violates coherence defining life as a paradox. We instead like to think of life as non-dually super-rational and prefer to define it by first distinguishing systems from meta-systems and then distinguishing the three special systems as a means of defining the autopoietic system as the balance point between the dissipative and reflexive.

We see in the Kleinian bottle the definition of inside and outside by the balance of two opposite Mobius strips. This is precisely the same as the balancing of complexnion vectors in the quaternion. However, when a symmetry breaking occurs the two complex vectors become a single quaternionic object that embodies the twisting of four dimensional space. Such twisting is hyper-efficient in that it prevents all knotting or blocking interference. Mobius strips conjunct to form a Kleinian bottle and they have the property that the inside and the outside surfaces are the same as well as being non-dual. There is no edge in a Kleinian bottle yet it defines a boundary with what lies beyond it. It also bridges via the circle of self-intersecting ambiguity into the fourth dimension. At the circle of ambiguous space the figure is at once open and closed to the outside world. It is the very definition of an openly closed system. Such a system has access to the outside world without breaking its boundaries. In the Kleinian bottle this ambiguity is embodied.

An autopoietic system has a set of nodes that produce itself. These nodes are controlled by a hyper cycle set in four dimensional space away from structural network of nodes. We can define an autopoietic system by noting that there are four kinds of patterning: value, sign, process and structure. The former are defined by Baudrillard in Critique of the Economy of the Sign and the latter are defined by George Klir in his chiasmic epistemological hierarchy (Architecture for Systems Problem Solving). When we move up from the level of pattern where Maturana and Varela use structure as the changing substrate of the autopoietic form we find that we can delineate the boundary of the autopoietic form using boundary algebra of the Laws of Form. This algebra allows us to delineate the form of the autopoietic system. However it ignores the "Laws of Pattern" which is the dual of the Laws of Form. Both Form and Pattern are needed to define the autopoietic system. The Laws of Form is a model of transcendence which ignores immanence. Both transcendence and immanence need to intersect in the Autopoietic form. What Stafford Beer calls the "muddy box" needs to be taken into account as well as the control structures that attempt to control it but ultimately cannot. We assert that it is necessary to use the Surreal Numbers as a model of the bifurcating structure that is projected down on the patterning and to serve as content at the level of abstraction where the Laws of Form are poised. We wish to include the "Laws of Pattern" as the dual of the Laws of Form which emphasizes the importance of nesting over multiplicity. These two exclusive viewpoints are like the duality of the Necker cube, their separation is illusory. We really need an algebra that includes both Form and Pattern. This can

be achieved by combining the Surreal numbers of Conway as the abstract content of the Laws of Form and also by realizing that there are also imaginary forms as well as real forms to contend with. We signify imaginary forms on the pattern of the quaternion by rotated marks. By that we situate the Autopoietic form in a four dimensional milieu. That four dimensional milieu is differentiated into the spacetime of process and structure and also includes a hyper-space of sign/value in which the hyper-cycle occurs which controls the self-production. The best model of the hypercycle is the five Hsing (transformations) of the Chinese. We note also that the model of the human body used in Acupuncture is basically autopoietic. They explicitly model the body as a series of channels in which solitonic Jing Chi flows which is self-maintaining and self producing. Within the four dimensional realm of the Matrix of spacetime and timespace we see the interaction between the controlling hypercycle and the nodes of self-production which in the case of Acupuncture are the points.

Finally we realize that each of the imaginary distinctions can be made with respect to the four aspects of Being: reality, identity, truth, presence. We would use August Stern's Matrix Logic as the means of representing the organizational level of the autopoietic system. Each of the fundamental aspects of Being may be used as a designation for the ordering of the organizational level. Thus we would have binary truth vectors in Matrix Logic for each of the aspects of Being. Logical operations would operate upon these truth vectors, reality vectors, identity vectors, and presence vectors and when these operations occur in the rings that August Stern describes they are autopoietic. The Matrix Logic describes the designation as Real, as True, as Present, as Identical of the Autopoietic System with respect to the meta-systemic environment.

The autopoietic system appears like a paradox when it is defined as Varela and Maturana do as the intersection between systemic organization and patterning structure. But if we instead use the distinction between the meta-system and the system and go on to define the special systems then we realize that the autopoietic system is better represented as something non-dual like the Kleinian bottle only dynamic. The definition of the autopoietic system as Maturana and Varela do is like the Necker Cube. It relies on nihilistic opposites for the definition which oscillate undecidably and then it jumps from the level of autopoietic form to autopoietic system magically by the introduction of observers from nowhere. In reality the autopoeitic system is itself reflexive and it mirrors the social reflexive realm within itself. As Onar Aam acutely pointed out on the Autopoietic Email List the very concept of 'organization' is social. We cannot really separate the dissipative and reflexive moments of the autopoietic system. It is a conjunction between the dissipative and reflexive.

Reflexive Special System

The next step up in our progression is the Hyper-Kleinian Bottle and the Tesseract. The Hyper-Kleinian bottle is the conjunction of two Kleinian Bottles. The Tesseract is the four dimensional analog of the Penrose Triangle and the Necker Cube. The tesseract is an actual

four dimensional polytope. It has eight three dimensional cubes all linked together though the fourth dimension. This figure is a reality in four dimensional space and thus breaks out of the illusion which the Penrose Triangle and Necker Cube incline toward. However, we run into a different problem at that level, which is that it is impossible to visualize directly the rotations of the Tesseract. Four dimensional space causes geometrical intuitions to fragment though lack of experience of that dimension. We can describe these figures algebraically but it is very difficult to reason about them geometrically. Thus, we can see the shadows of the Tesseract as it moves though our space, we can see the three dimensional impression it makes within our space, but we cannot see the actual relations between the cube that are connected in intersecting three-spaces that make up the tesseract. In this way the tesseract continues the progression of the Penrose Triangle and the Necker Cube. The tesseract as the four dimensional analog of the cube, casts its shadow on our three dimensional space but the figure itself is out of reach of our experience. Four dimensional space is incoherent when looked at from the third dimension and geometrically but from the viewpoint of its own dimensionality it is algebraically coherent. Incoherence becomes a matter of viewpoint and the underlying formalism one chooses as a basis for reasoning about the fourth dimension. Here we see that incoherence again returns at this level to be the fundamental property that is exemplified by the tesseract as it was by the Penrose Triangle.

The Hyper-Kleinian bottle is a conjunction of two Kleinian bottles in four dimensional space. To simplify the exposition we will create a construction in three dimensional space that is equivalent to the Hyper-Kleinian bottle. We do that by first noting that there are two basic representations of the self intersection of the three dimensional Kleinian bottle. One is the normal picture where the neck tube comes through the side and joins up with the hole in the bottom. But another completely different formation makes the self intersection a circle that goes round the whole bottle and makes the bottle look like a strange kind of torus. In this formulation we take a figure eight tube and rotate it 180 degrees and join the ends to form a Kleinian bottle. If we rotate the figure eight tube 360 degrees instead we get a Mobius strip. This formation produces a simpler and more elegant form for the bottle and Mobius strip. Now in order to get a Hyper-Kleinian bottle instead take a four leaf clover shaped tube where the four hollow leaves all intersect in a single line, then twist it 180 degrees to get a pair of Kleinian bottles wrapped around each other with the same circle of self intersection. If we rotate the same four leaf clover tube and join up the ends with a 360 degree twist we get intertwined Mobius strips instead. When we move to identify the four dimensional analog of the intertwined Mobius strips our attention is draw to the pentahedron in four dimensional space. The pentahedron is the analog of the three dimensional tetrahedron in four space. This geometrical figure is two Mobius strips intertwined without self intersection. In four dimensional space there is no need for the there to be any self-interference between the Mobius strips in order for them to form a complete figure. The pentahedron may

be seen as inscribed in a Hypersphere and in that Hypersphere there may be two dual pentahedrons inscribed in which the center of the faces of each are the points of the other. When we imagine this composite figure we are looking at the analog of the Hyper-Kleinian bottle. That bottle is produced from four Mobius strips that are conjuncted along their lines to form two Kleinian Bottles and these two Kleinian Bottles are conjuncted along their circles of self-intersection to form the Hyper-Kleinian bottle. That bottle may be seen as two pentahedrons arranged in complementary positions within a hyper-sphere. Or it can be seen as we have imagined it as two Kleinian Bottles intertwined except that in Four dimensional space each occupies a four-dimensional half-space (*anna* and *kappa*) on either side of a three dimensional space that slices the fourth dimension in half. In that three dimensional space is a sphere where the two Kleinian bottles self-intersect. The circle of mutual intersection becomes a sphere when we transfer the bottles to fourspace. This is because they may be oriented in any direction with respect to each other. If the singularity in the mobius strip is the line of its edge, and if it is the circle of self-intersection in the Kleinian Bottle then it is certain to be three dimensional in the Hyper Kleinian Bottle. This sphere of mutual self/other intersection is called the "sphere of ambiguity." That is because at that point of local incoherence it is impossible to tell if the bottles are intersecting self or other, and whether they compose one geometrical form or two, and whether there are one or two non-dual surfaces. Each of the pentahedrons contains five tetrahedrons. If we select any one of those tetrahedrons then they may be inscribed within a sphere. Two such tetrahedrons one from each pentahedron may be so oriented so that they are inscribed in the same sphere. If we look at the tetrahedrons we see that each is composed of six lines. Three of these lines in a kind of three dimensional Z pattern belongs to each Mobius strip. When we look at the two Mobius strip lines we see that what ever tetrahedron we choose within the pentahedron will become the sphere of ambiguity. When we align the two tetrahedrons so that the spheres they form line up we have defined a three dimensional sphere within the hypersphere that contains the two pentahedrons. That sphere of ambiguity can be seen as composed of the two independent circles that form the hypersphere. One circle is made up of the x-y plane and the other of the z-w plane. We can see ourselves as dynamizing these two unit circles and tracing out the two Kleinian bottles independently and simultaneously within the hypersphere. Where the two independent circles overlap we see the sphere of ambiguity. It has inscribed within it two tetrahedrons in reciprocal relation to each other (point for face). These two tetrahedrons embody the two twists of the two Mobius strips that make up the pentahedron in such a way that they overlap each other.

What becomes clear as we explore this strange geometry of the Hyper-Kleinian Bottle is that it is concerned with the interaction between three dimensional space and four dimensional space. Four dimensional space is composed of four three dimensional spaces in a quaternion relation with each other. Every three dimensional space is a slice of four dimensional space. It produces two half four dimensional spaces (*anna* and

kappa) on either side of it. And antipodal to the three dimensional space is a single axis that separates the two four dimensional half spaces at the other extreme away from the three dimensional slicing plane. This means that there are twelve virtual axes in four dimensional space. These are mapped by the quaternion twists into the four real axes of four dimensional space. We only see the virtual axes when we look at four dimensional space from the third dimension. If we use up three of the virtual axes in the three dimensional slice, and four in each of the four dimensional half-spaces on either side of the slice, then the final virtual axis appears antipodal to the three dimensional slice. As we will see shortly this is very significant. But if we look at the Hyper Kleinian Bottle we see that its main thrust is the connection between the three dimensional slice and the four dimensional container-space. The two Kleinian bottles each inhabit one of the two four dimensional half-spaces. They intersect in the sphere of ambiguity in the three dimensional slice. If we understand the Hyper-Bottle to be a three dimensional form involuted by mirror reflection then we can see the sphere as having two Mobius strip 'ears' which poke out into hyperspace. These ears reflect each other and return to the sphere. Or we can see the Hyper-Bottle as a bundle of spheres over a circle such that the spheres are mirrored though four dimensional space but intersect at the same circle. The dual Mobius strips can be seen as the two sides of the non-dual self-intersection of the set of spheres that twist around each other in four dimensional space. All of these geometrical formulations are equal. But the gist of their equality is that the sphere in the three dimensional slice can be looked at from either *anna* or *kappa* four dimensional subspaces. These subspaces can be thought of as mirrors that are reflecting the sphere from either the *anna* and *kappa* side. These sides are actually in all directions from three dimensional space. This mirroring is such that the sphere looks the same from either the *anna* or *kappa* side. There is no difference in the viewpoints. This is a very significant point. Viewpoints are unified non-dually in four dimensional space with respect to the Hyper-Kleinian Bottle. Looking at the sphere of ambiguity from the point of view of the Anna bottle is the same as looking at it from the point of view of the Kappa bottle. We can make a transformation from one viewpoint to the other seamlessly just as is suggested by the Lorenz transformation with respect to inertial planes in Relativity Theory. Mobius Strips make distinctions non-dual. Kleinian Bottles make the inside and outside non-dual and erase distinctions. The Hyper-Kleinian Bottle makes viewpoints non-dual and erases inside and outside. In the hypersphere any particular point may be on the inside, outside or on the surface of any three dimensional sub-sphere of the hypersphere. Inside and outside becomes ambiguous in four dimensional space. At its great circle boundary these three dimensional spheres shrink to points so that the hypersphere has a definitive boundary. But within that boundary and with respect to three dimensional representations or sub-spheres there is no telling whether a point is on the surface or on the inside or on the outside of the hypersphere. The inside/outside distinction becomes ambiguous except for the great circle outer surface of the hypersphere. When we lose our distinction between inside and outside

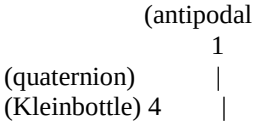
we gain the non-dual fusion of viewpoints. All the viewpoints are merely the vectors of four dimensional mirroring. In this mirroring the hypersphere looks like a bundle of spheres that get smaller and smaller till they become points at the great circle boundaries. Within that boundary there is a fractal arrangement of interfering spheres that are mirrored in all directions to infinity. Each vector viewpoint from the fourth dimension makes the fractal sphere look identical. The same is true of pentahedrons inscribed in the hypersphere. They form a fractal arrangement that mirrors off in infinity from what ever direction you look at it in four dimensional space. Each pentahedron has a dual in the orthogonal four dimensional half-space. Similarly all the three-spheres of the hypersphere have a dual that also appears to be mirrored to infinity getting smaller and smaller as it approaches the points on its hyperspherical surface.

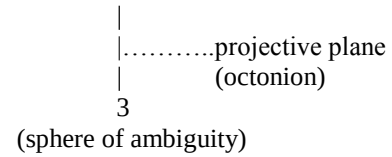
This non-dual fusion of viewpoints in four dimensional space is the fundamental basis of the socius. Within the social field there is a non-dual unification of all the individual subjective viewpoints prior to their individuation. This is the underlying basis of the social and is what allows us to have reflectivity. The four dimensional realm fragments the views and produces 720 degree mirroring in all directions but at the same time makes the viewpoints non-dually fused, so that there is a single non-dual viewpoint that appears to be diverse in lower dimensional slices which we have access to. Thus in the fourth dimension coherence and incoherence become the same thing. Similarly duality and non-duality become the same thing. Steve Rosen calls this non-dual duality (or we may say chiasmicly dual non-duality). The tesseract is from one perspective coherent and from another incoherent. So it is with the Hyper-Kleinian Bottle. It is from one perspective dual and from another non-dual. It is dual in that it is composed of two different dual mirroring Kleinian Bottles. But it is non-dual in that those two Kleinian bottles are conjuncted at their self-intersection point so we cannot tell them from each other and so it becomes ambiguous whether they are one or two. The tesseract is super-rational while the Hyper-Kleinian Bottle is paradoxical in this case. So we see that the set of figures in the series Mobius, Klein, hyper-Klein interchanges places in the fourth dimension with the series Penrose, Necker, Tesseract with respect to the embodiment of super-rationality and paradoxicality. Thus we see that paradoxicality is within super-rationality and vice versa if we go deep enough.

This complementarity extends to the structure of four dimensional space itself. Four dimensional space is as we said four three dimensional spaces in a quaternionic relation to each other. In the Hyper-Kleinian bottle we see the relation between the four dimensional half-spaces either side of the three dimensional slice. The sphere of ambiguity, localized incoherence, resides in the three dimensional slice. That sphere looks the same from either the *anna* or *kappa* side of the three dimensional subspace (hyperplane). All viewpoints on the sphere are non-dually fused in hyperspace. Viewpoints themselves are like the dual mirrored Kleinian bottles that appear on the *anna* and *kappa* side of the subspace. The subspace takes up three of the twelve

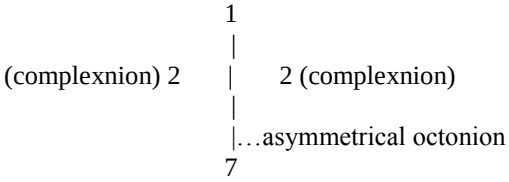
virtual axes of fourspace that appear when we consider it to be composed of four three dimensional spaces. Eight virtual axes are taken up by the two four dimensional subspaces. That leaves one antipodal axis to the threespace. The combination of this antipodal axis and the threespace is a projective plane. The projective plane is the dual of the Hyper-Kleinian Bottle. They are complementary. The Hyper-Kleinian Bottle is ambiguously dual made up of lower dimensional non-dualities. The Projective Plane at the same level of abstraction is non-dual due to its non-orientability for lower dimensional figures in threespace. The two four-dimensional half-spaces contain Kleinian bottles that define the sphere of ambiguity. The sphere of ambiguity defines a threespace slice out of fourspace. The threespace slice is connected to the antipodal axis to create a projective plane that is non-dual at the four dimensional level without ambiguity. The antipodal axis when taken with all the other antipodal axes of the constituent threespace slices of fourspace defines the unification of all possible viewpoints into a single non-dual social viewing manifold.

When we realize that the structure of fourspace is defined by the relation of the projective plane and the hyper-Kleinian bottles then we recognize that this is the dual of another complementary configuration of fourspace. That complementary configuration would divide up the virtual axes differently. Instead of dividing them up so:





it divides them us instead in this way:



When we look into these two patterns what we see is very significant. One configuration of fourspace virtual axes defines the relation of the asymmetrical quaternion to the symmetrical octionion while the other defines the relation of the asymmetrical octonion to the symmetrical quaternion. Here the imaginaries are the virtual axes of fourspace themselves. So the complementarity of duality and non-duality brings us back to the quaternion which represents the autopoietic and the octonion that represents the reflexive. Reflexivity is embedded within autopoiesis and autopoiesis is embedded in reflexivity. The asymmetrical configuration of the quaternion is related to the symmetrical octonion and vice versa. In the latter figure seven dimensional space is antipodal and produces an analog to the projective plane. This is orthogonal to two dimensional spaces which embody the complexnion. We may think of these TWO two dimensional spaces as articulations of the mandelbrot set. They intersect to create a quaternion mandelbrot. The opposite of these is an octonion mandelbrot first pictured by Onar Aam based on a hypothesis of the author. We further hypothesize that the quaterbrot and the Aambrot (octobrot) sets are the dual of the Hyper-Kleinian

bottle and the Projective Plane. They form a Yin/Yang formation equivalent to the Great Ultimate in which on one hand we have infinite fractal sets that intertwine and on the other we have the Hyper-Kleinian and Projective Plane that reconcile duality with non-duality and coherence with incoherence.

When we put these geometrical excursions in the context of the Reflexive we see that a peculiar picture arises of what Onar Aam hither to called the Mirrorhouse. In the mirrorhouse of the social there is infinite mutual mirroring which underlies all social interactions. We see this in the progression of mirror configurations from the mutual facing mirrors analogous to the complexnion and embodies the underlying dissipative structures, the three triangularly facing mirrors analogous to the quaternion embodies the autopoietic structures, and the inwardly mirroring tetrahedron that is analogous to the octonion embodies the reflexive structures. The reflexive level of mirroring produces an enclosed reflexive space. This is the analog for the sphere of ambiguity. It is ambiguous space from the viewpoint of surrounding flat space. But internally it is an enclosed mirroring surface. The space enclosed by that mirroring surface has the nature of the Palmer Chiasmic Cube of self intersecting non-dual reversibilities. It is a non-dual field that borders on absolute non-duality of interpenetration. In it every viewpoint has an opposite that it can be transformed into and all viewpoints intersect at a single point that is a singularity which we call following M. Henry the Essence of Manifestation. The Hindu's called it *Atman*. Hegel called it Spirit. It is in the social non-dual viewpoint manifold as the point where all the projective planes intersect. It is the opposite of the sphere of ambiguity. It is non-local global coherence of all viewpoints. The seeming fragmentation of the socius as social field is based on this higher dimensional non-dual fusion of viewpoints. This is what makes resonance between desiring and disseminating machines possible.

Reflexivity occurs within the mirrorhouse. All the paths of lightrays though the mirror house are non-dually fused into a single higher dimensional path via Projective Plane. All the viewpoints within the Mirrorhouse are non-dually fused together via the Hyper-Kleinian Bottle. The views of two bottles on the same sphere of ambiguity are the same. The reflexive space is fragmented by the series of hyper-mandelbrot sets despite its higher dimensional unity. So the mirrorhouse is infinitely deep fractally and infinitely complex chaotically. Asymmetrical quaternions are embedded in octonions split into symmetrical quaternions. Asymmetrical Octonions are embedded in quaternions split into symmetrical complexnions. This dual embedding structure gives us the complemetrarity between the autopoietic reflexive and the reflexive autopoietic. The autopoietic special system has a reflexiveness in the organization of its self-producing nodes. The reflexive special system which is intrinsically social is made up autopoietic individuals. The interior of the autopoietic system mirrors multidimensionally the exterior of that system where multiple autopoietic systems resonate and interact. This is an image of the mirroring of interpenetration.

Paradoxicality and Super-Rationality

In our unfolding of the roots of Mathematics below the surface of the Pure Presence Being we have uncovered some anomalous mathematical entities that we have used as a basis for modeling the transition between Systems and Meta-systems. We recognize all the categories of mathematics as systems and we see them in the field of all possible categories which is their meta-system. Mathematics does not thematize the meta-system that encompasses the categories. When we do thematize it we find that the transition into that meta-system can be modeled by a series of emergent stages and those emergent stages can be seen mirrored in the series of topological anomalies or the series of hyper complex algebraic anomalies or even in the series of paradoxical figures. So from this vantage point we envision functors that go between the different series of anomalies that give us different ways to talk about the same three emergent levels between the system and the meta-system.

But in some private discussions with Steve Rosen on the Octonion Appreciation Society email list it became clear that there was an important difference between the topological series and the algebraic series which can only be seen when we take into consideration super-rationality and its relation to paradoxicality. Super-rationality as we have said appears in Koans of Zen Buddhism. They are statements that take us beyond what is rationally comprehensible. But Nagarjuna gives us an even better way of thinking about the super-rational. He uses Indian Logic as his basis. Indian Logic does not accept Excluded Middle which is Aristotle's prime dictum. Indian Logic posits four statements: A, ~A, Both and Neither. Nargarjuna shows us that Emptiness is the difference between the Both and the Neither. Emptiness is super-rational. That is why Koans may indicate enlightenment as they specify the languaging that might occur at the point of entry into enlightenment, which is merely the realization of emptiness as the fullness of interpenetration. When Steve Rosen and I discussed the topological series and its non-duality I warned him that it expresses the super-rational just as much as the paradoxical which he had emphasized in his works. He said that he did have in his though a place for what was not paradoxical which was the Leminscate which is what you get when you cut the mobius strip in half, i.e. a two sided band which is twisted twice. This made me realize that there was a good chance that the Leminscate represented the Super-Rational and that the other forms in the topological series were devolutions from that toward paradoxicality. The fact that the Hyper-Kleinian bottle defines the sphere of ambiguity enforced this realization.

Now what we realize is that actually the topological series is the reverse of the algebraic series in terms of the definition of the movement from System to Meta-system. This means that the topological series has another level of correspondence with the algebraic series which is the opposite of that outlined above. In this correspondence it is the Hyper Kleinian bottle that is related to the dissipative system and the mobius strip that is related to the Reflexive system. We can realize how this is so if we think of the series of paradoxical

figures (Penrose, Nekker and Tesseract) as the outward appearances and that the topological series gives us the inward realities associated with these appearances. Thus the Hyper Kleinian Bottle defines the implicit reflexivity of the dissipative. This is to say that the structure of the dissipative system combines a circuit of Logos with a circuit of Physus into a single loop that appears outwardly to act like the Escher waterfall. But this outward difference between the loop of logos and the loop of physus becomes one though the inner possibility of the Hyper-Kleinian Bottle. It is that which makes the singularity and the boundary of the dissipative system one yet not one at the same time. As the reflexive properties become externalized then we see the over-spilling of two dimensional paradoxical appearances into the four dimensional tesseract. The inward possibilities become simpler so that we see within the reflexive system only the non-duality of the mobius strip that is one step away from complete interpenetration in the Meta-system.

If we realize that the two series are reversed with respect to each other from a certain point of view then it becomes clear that the topological series is telling us something different from the algebraic series. When we consider what that might be then we realize that if we take a pair of autopoietic systems in a reflexive milieu that they must continually define their own and each other's boundaries. This self other definition that occurs within the reflexive system is governed by the topological series. In other words it becomes clear that the definition of the boundary may range on the spectrum from super-rational to paradoxical. At the level of the paradoxical the whole reflexive system becomes the sphere of ambiguity. At the super-rational level there is a clear distinction between the two autopoietic systems which is at the same time their own distinctions and the other's distinction. In other words at the super-rational level distinctions are mutual resonances. Between these two ends of the spectrum the distinction goes through a series of devolutions from the superrational through the mobius strip and the kleinian bottle and the hyper-kleinian bottle. At the first stage the clear distinction becomes non-dual in the mobius strip, i.e. locally dual but globally non-dual. Then it devolves further to the point where the distinction is lost in the Kleinian Bottle but the inward and outward becomes non-dual. Finally the circles of ambiguity of each kleinian bottle, i.e. each autopoietic system, becomes intertwined. This produces the hyper-kleinian bottle configuration where the self cannot distinguish itself from the other. Finally this degenerates into the sphere of ambiguity where there is perfect paradoxicality. If the sphere of ambiguity itself involutes then it becomes meta-paradoxical or absurd.

This picture of the autopoietic system producing its distinction with respect to another autopoietic system in the reflexive environment and the devolution of these distinctions in a quantal series gives us an even more precise model of the special systems in their relation to each other. The difference between the topological series and the algebraic series is an important discovery that has far reaching implications for the modeling of autopoietic social systems.

Wavicles and the Emergent Meta-

system

Solitons are solitary waves that act like particles. We might call them wavicles. These non-dual waves can be seen to play an important role in the definition of dissipative, autopoietic and reflexive systems.

Besides the topological series, the algebraic series and the series of paradoxes there is another important anomalous series that must be considered. That is the series of Soliton, Breather and Super-Breather. Solitons have long been posited to be the model of super efficiency that gives us a physical representation of the dissipative system. It is known that Soliton waves form Breathers when a Soliton and an Anti-soliton (hole) interact. This causes a stable structure where the soliton falls continually into the anti-soliton and the anti-soliton falls into the soliton. This gives us a 'breathing' motion that is static in space. We have long thought of this as a analogy for the autopoietic special system. Also we have had the hypothesis that there is a super-breather that is the conjunction of two breathers that corresponds to the reflexive special system. This hypothesis was strengthened when the form of the Hyper-Kleinian bottle was recently discovered by the author. This led to the postulate of the form of the Super-Breather. Solitons have another form which is called the Instantaton. The instantaton is a soliton that pops from point to point by moving though troughs in the potentials of a field. They seem to move across spacetime instantaneously without passing through the intervening space. They merely pop from one place to another. There is a theory that electrons and other particles are really instantatons that pop around to create their statistical shell around the atom. In this scenario electrons do not actually move. Instead they vanish one place in their electron shells and then momentarily appear somewhere else in the shell. This process repeated over and over gives us the image of an electron moving around its shell.

I combine this idea of the instantaton form of the electron to create the idea of the Super-breather. The super breather is composed of two breathers separated in spacetime exchanging solitons via instanation jumps. Such a formation would give us just the kind of properties that we would expect at the reflexive level. It would give us information and energy exchange effortlessly at a distance between autopoietic nodes. Thus if each autopoietic node is a breather formation then the super breather is a means for each node to participate with the other nodes in the network exchanging information and energy across the intervening spacetime within the autopoietic system. A similar kind of exchange may be taking place between autopoietic systems within a reflexive system. This ability to exchange information and energy at a distance between breathers can explain how the autopoietic system can be so stable and energy efficient. It can also explain how it can achieve its self identity as a basis for self production. What Jahn and Dunne call quantum tunneling between consciousnesses may be exactly this kind of super-breather sort of information and energy exchange. We know that the nodes within the autopoietic system must be an image of the whole of the autopoietic system itself. We think of these nodes as a kind of dynamic hologram where each part

mirrors all the others and this is how self-identity is achieved. But the super-breather concept allows us to understand the dynamic resonances that underlie self-identity of the autopoietic system.

Here again we can see how another anomalous series can give us new information about the special systems. In this case we are learning not about the distinguishing of one autopoietic systems from another but about the dynamics of action at a distance where there are exchanges of information and energy in quanta across spacetime. This ability explains how the autopoietic nodes exchange information necessary to establish their self identity although they are all in different places. It also explains what the quantum tunneling between consciousnesses described by Jahn and Dunne[1] might consist of at the social level.

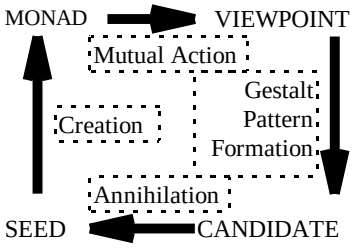
The nodes of the autopoietic system may be seen as a swarm of monads that appears in the Emergent Meta-system (EMS). Super-breather instantaton hops might be considered the means of moving from one EMS phase to another. Thus we might consider that the nodes of the autopoietic system move though an EMS cycle from seed to monad to viewpoint to candidate as the means of establishing its self-production. Mutual action between the autopoietic nodes may be via the instantatoin super-breather soliton exchange. Gestalt pattern formation may be though the process of establishing distinctions governed by the topological series. Annihilation and creation occur when the autopoietic system pops into and out of (apoptosis) existence. Or alternatively we can see the illusory continuity of the autopoietic nodes as being produced by continual creation and destruction in very small time quanta.

The seeds of the autopoietic system are the memories of the prior period. We access those memories and we produce a picture or vision of the self of the autopoietic system. The nodes then attempt to reproduce this vision or picture. They each have their own viewpoints on the rest of the swarm of autopoietic nodes. They produce candidate lists based on their mutual action to survive to the next cycle. Then the candidate lists annihilate until there is only the seeds for the next cycle and those seeds become the memories from which the new EMS cycle will arise.

The EMS cycle is produced out of the special systems. Each special system is a meta-operator within the Emergent Meta-system.

Real System = creation
Dissipative System = annihilation
Autopoietic System = mutual action
Reflexive System = gestalt pattern formation
Meta-System = Emergent Meta-system

These form a cycle:



This cycle solves the problem of how we can have radical emergence without causation. This is a fundamental problem posed by Mahayana Buddhist metaphysics. The EMS cycle solves this problem by positing that there is an light

[1] Jahn & Dunne Margins of Reality

bouncing off a inwardly mirroring tetrahedral surface. Conway showed that there is one such path that allows a ray to return to its origin within a tetrahedron. The EMS cycle follows this path and bounces off the mirrored surface producing a cycle in which there is a transformation each phase of the cycle that moves though the series from the System though the special systems to the meta-system. In this way the meta-system continuously reproduces itself non-causally across time. This model is similar to Ben Goertzel's Self-Generating "Magician" System model except it allows for creation ex nihilo. We can construct the same model with instantaton jumps between breathers in a super breather if we consider each autopoietic node within an autopoietic system to be a breather. Breathers are continually annihilating and creating themselves as the positive soliton falls into the negative soliton and vice versa. The super-breather formation allows this to occur across spacetime in such a way that information and energy is exchanged. This exchange may be seen as an image of the Emergent Meta-system formation.

Disipative Autopoietic Reflexive System Modeling

In this section we will use the information from the different series of anomalies to construct a model of the Autopoietic System in a reflexive environment. This model makes use of a new realization that there is an inherent relation between the minimal method viewpoints, the aspects of Being and the kinds of patterns.

The aspects of Being are as follows:

- Real
- True
- Identical
- Present

It has been shown by Butchvarov[2] that material identity implicitly assumes that there are pre-entities that are identified. We can see from this that each kind of pattern has an inherent relation to a particular aspect of Being:

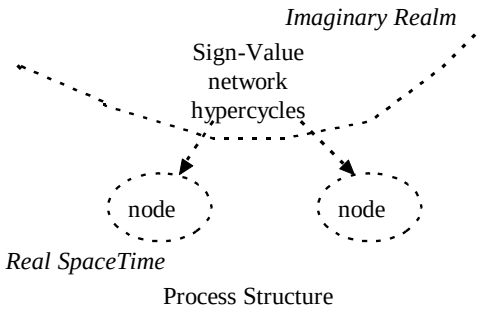
Value = Real
Structure = Identity
Sign = Truth
Process = Presence

This structural ontological model that posits pre-entities gives us a basis for projecting the emergent level of pattern below that of form. Patterns are the content of forms and this content comes in four types. Klir has described process and structural kinds of pattern in Architecture of Systems Problem Solving. Baudrillard has described the sign and value kinds of pattern in Critique of the Economy of the Sign. All four of these may be seen to follow the pattern of the epistemological hierarchy posited by Klir. They all give rise to meta-levels of patterning and also the different kinds of pattern entwine chiasmically.

What we like about this formation is that it makes possible a basis for describing the autopoietic system. That system has a control hyper-cycle that exists in an imaginary "space" over and above the spacetime in which the autopoietic nodes appear. The kinds of pattern allow us to describe that imaginary "space" in terms of sign/value as opposed to the real

[2] Butchvarov, Panayot Being Qua Being Indiana University Press

space produced by process/structure. Process is the meta-models that dictate when structural changes in our systems model must change. Structures dictate the patterning of values of variables. They are like functions over those variables. The Process meta-models dictate the circumstances when the functions or the linkage between functions change for a given variable. Together they give a model of the spacetime changes in a system. But sign and value may be seen as nowheres that are the basis of producing process or structure based dissipative systems. Sign is a no-when and Value is a no-where. Sign and process create a dissipative structure and Value and Structure create a dissipative structure. When these two dissipative structures come together they create an autopoietic system. In that autopoietic system sign and value may be seen to define an imaginary space over the spacetime of the process and structure pair. In that imaginary space sign-value pair vectors produce a network within which hypercycles can form. These hypercycles are the control structures for the autopoietic nodes within the autopoietic system.



Now here is an interesting point. Once the sign-value hypercycles form then the Processes and Structures can relate to either Value or Sign independently. This produces the four viewpoints on any real-time system which has been previously identified in Wild Software Meta-systems by the author.

- Process Sign = event
- Process Value = function
- Structure Sign = agent
- Structure Value = data

Once we have realized this connection then everything that is known about the four methodological viewpoints comes into play within the autopoietic system. We may posit that the boundary between the imaginary realm of sign-value and the spacetime realm is like awareness. What is controlled by the hypercycle exhibits intentionality and thus is directed awareness or consciousness. What is not controlled are merely statistical processes and these are unconscious. This also helps us define the difference between organization and what Maturana and Varela call structure. Organization is what is intentionally controlled by the hypercycle while structure is the patterning that goes on in the spacetime realm that is not controlled explicitly in the process of self production. This also allows us to relate this model to Jungian psychology which defines the self as the totality of the conscious and the unconscious. The ego is the limited identity that is totally self-conscious. The ego exists in the Logos of the imaginary realm. The self includes the spacetime embodiment as well as the ego. The self is the timespan it takes the ego to become itself. It is the quantum of time necessary for it to produce itself again out of itself. Awareness is the non-dual man between the conscious and unconscious.

The four viewpoints give rise to the sixteen minimal methods. The

viewpoints appear right at the boundary of awareness. Thus the sign-value networks may be seen in terms of petri net and state machine control structures. We note that for the monads there must be a transformation into viewpoints in the EMS cycle. We posit that the arising of these relations between the kinds of patterns are the means of by which the monads archive their viewpoints on themselves.

Given this insight it should be possible to reconstruct the actual EMS cycle that the autopoietic nodes participate in. We note that there is a sharp distinction between the sign-value networks and the process-structure spacetime in which the autopoietic nodes exist. It might be possible to see the seeds of the EMS cycle as the sign-value pairs in the hypercycle networks and the activation and deactivation of nodes as the creation and annihilation meta-operators. Now if this works then it is possible to see the hypercycles as the seeds and the autopoietic nodes and the monads. The seeds activate and deactivate the nodes and that corresponds to their creation and destruction. The nodes form a swarm of monads. These monads engage in mutual action when they are activated. Out of that mutual action comes the arising of viewpoints. The viewpoints arise though the interaction between the sign-value and process-structure arenas. Process can relate either to sign or value as can Structure. The sign-value networks get interpreted as petri nets or state machines. The monads are viewed in terms of their autonomy as agents, their functionality as transformers, their data and events which intertwine in the self-production process. Self production is seen as the production of candidates for propagation to the next cycle of the EMS lifecycle. This is done by each monad producing an internal model of its relations with the other monads based on its experience in the mutual action phase. Those models are completely internal and there is no relation between the separate models that are produced. However we can have a fuzzy summary externally of the different posited relations. The monadic autopoietic nodes have the form of computational monads. Computational monads are constructed out of the aspects of the minimal methods that do not require any relation between design elements externally. We are thus moving from Laws of Form to Laws of Pattern as the monads develop a layered model of their posited relations with other monads in the swarm of autopoietic nodes. Out of their separate models the monads posit what other elements should be in their solipsistic design. These elements are drawn from a structural basis in which virtual monads and virtual anti-monads are produced. Different monads select from this structural virtual soup those it would like to include in its design. The design may take the form of a non-well-founded hyperlist (instead of hyperset). This is kind of like the different buildings that MUD users create independently of each other. Each user creates the rooms and furniture he or she wants in their virtual environment adding on to the already built maze that others have created. Mud users can destroy the work of others by not allowing them to connect their buildings to the already existing building. Each user decides what it will allow to be connected to his own buildings. Thus the rhizome of the buildings in the MUD is controlled by each independent user allowing or disallowing connections. Now we can see a similar structure in the internal

design of the computational monads that are the autopoietic nodes. Each monad creates the design of the swarm using the minimal methods as it sees fit. Each uses structures of monads that are drawn from the structural pool which are opposites. When we take these various structures produced by each monad as a hyperlist and place them together then the elements and anti-elements cancel each other out. This leaves the seeds for the next life cycle phase. Now the question is how this EMS cycle can be the basis of self production.

Self-production occurs when there is interaction between the autopoietic nodes that are embedded in process and structural patterning and the network of sign-value pairs that make up the hypercycles. In this interaction the first moment is the activation of certain autpoietic nodes and the deactivation of others. This inhibition and excitation by the hypercycles produces a kind of cancellation because inhibition and excitation by different hypercycles cancel each other. Given a set of autopoietic nodes that are turned on then they develop viewpoints from the interaction of process and structure separately with either sign or value. This produces the agent, function, event and data viewpoints. Out of the interaction of these viewpoints arise the minimal methods. The sign-value pairs get interpreted as petri-nets or state machines and these allow control to occur in a fashion like software programs. All the minimal methods may be employed first to define the nodes as computational monads and then to build up inside the layering of each one an image of the design of the swarm where each node posits the architectural connections to all the other nodes that it sees as important. These architectures may be non-well-founded hyperlists where monads can be members of their own hierarchies and where set members may be repeated many times in different contexts. Thus each monad develops a vision of the whole swarm of autopoietic nodes. They can communicate about their visions and share information though mutual action. This mutual action as we have seen is probably carried though super-breather exchanges of instantons at a distance. Externally we can only see fuzzy summaries of the overall structure of the swarm architecture. What is interesting about this is that it is possible to apply genetic algorithms to the production of the architecture. We merely produce swarm members randomly which have different internal models. The fitness functions are applied to the fuzzy summary of the architecture which then applies evolutionary pressure to the individual monads. Now this fitness function allows us to select which monads will be excluded from the swarm based on the overall fitness of the swarm and the individual monads contribution to that fitness. Thus we can imagine two different forces acting in self production. Externally we can see genetic algorithms and internally the cancellation of the individual designs of the monads. Self production is a tension between these two different factors. Self-production is seen as the evolution of the swarm where the self is the essence of the species that is produced by crossover and mutation. Cross over occurs when hyperlists are shared. Mutation occurs when a new monad is produced with a randomly generated hyperlist of swarm design. But internally Self-production is the collusion between nodes that allow particular nodes to survive from one

lifecycle to the next. Ben Goertzel describes this collusion as the social basis of Self-Generating Systems. Self Generating Systems assume discontinuity rather than continuity as the basis of the meta-system. It is this assumption that makes the Autopoietic System a partial meta-system or a broken and fragmented system rather than a whole with continuity. Yet the Autopoietic partial meta-system can still produce itself. It does this by the collusion of the Autopoietic nodes in the EMS cycle. Each one persuades the other not to cancel it out by including it in their hyperlists rather than the its structural opposite. We see here that structuralism is the basis for cancellation. Process on the other hand is what changes out structures over time. Process is what produces the segmentation in time of the EMS lifecycle that is occurring inside the autopoietic system. Signs are the basis of all reference within the Autopoietic system and also self reference. It takes two sign-value pairs to point back at itself and produce a minimal hypercycle which merely cycles back into itself endlessly. Value is what allows the hypercycles to set minimums and maximums on a particular parameter and thus allow control of that parameter. There must be a network of parameters that represent the organization of the Autopoietic system. This network is used as an envelope of constraints that governs when certain nodes are turned off and on. So self production occurs in terms of external fitness and internal cancellation and collusion. By self we mean the totality of the self including awareness, intentional consciousness and the unconscious. Each autopoietic system has as Jung says a shadow, an animus and anima, a wise old man and crone and an archetype of the Self as a totality. The shadow is the anti-swarm to any specific swarm signature lifted out of the structural soup of nodes and anti-nodes. The animus-anima is the mirroring of the anti-node in the node and the node in the anti-node. The wise old man/chthonic female crone is the next meta-level mirroring of node-anti-node or anti-node-anti. Finally the archetype of the Self is the EMS structure itself. Each of these mirrorings take us up one level in the hierarchy of Hyper-complex algebras. Each mirroring shows us the depth of time where the nodes look into their future or past and see that in the generations of other nodes that surround them. Like the forest that has trees of all ages the nodes can look at other nodes that are at earlier and later stages of their lifecycle within the EMS lifecycle. Self Production must have a genetic development structure over time governed by Process. What the self is that is being produced is continually changing as the essence of the individual organism unfolds in their genetic development. Self production is the production not of the identical but of the Same. This sameness can be produced because of the Belonging Together of the autopoietic nodes in the swarm. Their belonging together inwardly is a collusion in which the group allows the individual to survive from cycle to cycle. But outwardly the belonging together is their mutual filling of a niche in the environment and their adaptation to that niche by the response to selective pressures.

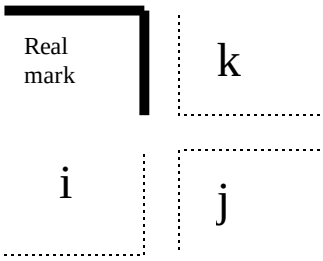
This concept of the autopoietic nodes as an EMS cycle is a very sophisticated vision of the Autopoietic System. We already know that the Autopoietic nodes are themselves autopoietic systems.

They have a quaternion formation that makes them like mirrors that reflect the rest of the swarm. The swarm of autopoietic nodes has the same relation to each other as the set of autopoietic systems have within the reflexive social environment. There is a social environment inside the Autopoietic system that reflects the Social environment outside it. This refection between inside and outside makes the Autopoietic System istransparent even though it is informationally closed while energetically open. So we can see that the autopoietic system is awash in a reflexive social sea but it also has that reflexive sea inside itself. Now we realize that the autopoietic system and the autopoeitic nodes that make it up BOTH achieve their self production by participating in an Emergent Meta-system cycle. There are two EMS cycles going on both on the Macro and Micro scales at the same time. By the micro cycle the Autopoietic system produces itself. By the macro cycle the Society of Autopoietic systems produces itself. These two processes mirror each other so that we can as Socrates suggests in the Republic look into the soul of the individual by looking at the way that the city works. Self Generating *meta*-systems or Emergent Meta-systems as I call them produce the self as a totality as Jung suggests and are not merely the perpetuation of the self conscious and self identical ego. This is possible because all the aspects of Being (reality, identity, presence, and truth) interact chiasmicly though the kinds of patterns that exist at the pre-entity level. This interaction produces the viewpoints (agent, function, event, data) which in turn allow the minimal methods to arise which then form the basis for the internal designs of the autopoietic nodes that eventually participate externally in a genetic algorithm and internally in the cancellation process that allows for collusion.

In order to specify more precisely what we have said generally let us proceed to talk about the steps of the arising of the Emergent Meta-system as the means of Self-production in the Autopoietic system. First the general vision is that we have at least two autopoietic systems within a reflexive environment. The autopoietic systems have forms governed by the quaternion algebras and the reflexive field is governed by the octonion algebras. Within the autopoietic system we follow Maturana and Varela by seeing a set of autopoietic nodes that together somehow manage to produce the autopoietic system by working together, i.e. socially. Now what we are positing is that these nodes are members of an swarm and that they self produce in the form of a Self Generating System (or Emergent Meta-system). This means that the form of the meta-system produced by all the meta-operators that come from the entire set of special systems work together to create the self-identity of the autopoietic system over time. This is a radical concept of how self identity is produced because it is based on the assumption of radical discontinuity instead of continuity which is the usual foundation of systems theory. We are saying that the Autopoietic Special system is really a partial meta-system and a fragmented system and thus falls between the System and Meta-system. Thus, we cannot assume continuity as we would in a normal systems theory. Therefore, we must assume discontinuity and explain continuities rather than the other way around. Self-production is no-longer a

creation of identity, but instead something more like the *Sameness* that Heidegger defines as *Belonging Together* in Identity and Difference.

Now how do we approach understanding the Autopoietic nodes though the Emergent Meta-system formation as a means of self production? We begin by saying that the autopoietic special system is caught in Being as an eventity. It thus has the same aspects that Being has answering to truth, reality, identity and presence. Truth and Identity reflect the Logos and Reality and Presence reflect Physus. So the autopoietic system has an appropriation to each of these aspects of Being. This shows up when we consider the level of pattern. The level of pattern can be identified to be composed of pre-entities that are appropriate to the four aspects of Being. These pre-entities are of four kinds: process, sign, structure and value. Sign is related to reference, Value is related to assignment of a signature out of a typeset, process is related to the changing of structures over time, and structure is related to the actual bit level coding below the typing super-structure. Each of these kinds of pattern defines a kind of pre-entity related to the aspects of Being. Before we can talk about autopoietic systems we must first talk about autopoietic forms that exist as an emergent ontological level between pattern and system. At the level of form we want to consider G. Spencer-Brown's laws of form as a reference formalism as Varela does in his work on dynamic forms with Kauffman. We see the kinds of pattern as the contents that fills these forms produced out of 'marks'. Because G. Spencer Brown's marks are both operators and operands we consider them autopoietic forms. That is to say they are forms that are both nouns and verbs at the same time. An autopoietic form is one where ***form forms*** *itself.* We think of this process of form forming itself on the model of the inter-transformations of the quaternion. This means that we need imaginary marks. We produce these imaginary marks by rotating the normal marks used in normal Laws of Form



These marks transform into each other according to the algebra of quaternions. Due to this self-identity has a very precise model. There is also a precise interface between the imaginary realm of marks and the real marks. Given the laws of form we can see that this allows us to model three of the four kinds of Being. Pure Presence Being is modeled by the vertical staves of the mark. Process Being is modeled by the horizontal roofs of the marks. Hyper Being is modeled by the jumps backward and forward from different points in the nested marking structures. What is missing is Wild Being which relates to the contents that the marks surround. We now know that these contents considered as pure non-manifesting Hyle appear in four kinds. We want a simple way to articulate these kinds of patterning of contents. We do that by introducing the formalism of the Surreal Numbers. Each kind of pattern is represented by a different style of up or down arrow. The concatenation of the

arrows of a particular style pattern produces a surreal number. The surreal numbers can fill the marks and give us a complete model of the kinds of Being which also represents all the aspects of Being at the same time.

(/\ /\ /\ /\ \/ \/ /\ \/ \/ \/ /\)((\/ /\ /\ \/) /\ /\ )

This is a pattern of surreal arrows in a laws of form mark sequence. In order to fill this out we would have to consider four kinds of arrows that relate to the kinds of pattern (sign, value, process, and structure) and four kinds of marks that represent the three sorts of imaginary marks. This gives us an unified model of Being in terms of kinds and aspects.

But there is another consideration. Laws of form is contrasted to what I have previously called Laws of pattern that has opposite laws:

Laws of Form = *RULE*
()() = ()
(()) = null

Laws of Pattern = *ANTI-RULE*

()()= null
(())=()

Laws of Pattern are not considered by G. Spencer-Brown. They are a model of immanence instead of transcendence. They are a model that affirms layering instead of multiplicity. Thus we need to be able to switch back and forth between laws of form and pattern. It has been suggested that we do that by introducing braces and saying that what ever lies outside the braces are laws of form. However the layers of braces allow us to pass back and forth between laws of pattern and laws of form at will in the nesting of the two sets of laws

((/\ /\ { (\/ \/ /\ /\ ) (/\ ) } \/ \/) /\ /\ )

What comes out of the realization of the duality between Laws of Pattern and Laws of Form is the fact that there are really four elements that both kinds of Laws deals with namely:

- Form
- Nothing
- Layering
- Multiplicity

Laws of Form and Pattern give different priorities to either layering or multiplicity. However, there is a fundamental misconception of Spencer-Brown that needs to be cleared up. He identifies the null with the ***void*** or ***emptiness*** from Eastern religious philosophy. This is wrong. The null is just an empty background. We only get to the void or emptiness if we posit an anti-form. When we posit an anti-form then we can say that something is both the form and the anti-form or we can say that something is neither the form nor the anti-form. Emptiness or void is between this both and neither. Thus we must have the possibility of including para-complete and para-consistent logic in order to approach emptiness. In order to accomplish the we will introduce Matrix Logic of August Stern. This logic allows truth vectors that include **both** and **neither** truth values. However, we cannot apply these just to truth but must apply these to all four aspects of Being. So we will have in addition to truth vectors also reality vectors, presence vectors, and identity vectors. It is though this means of introducing truth, reality, identity and presence vector subscripts

and treating these with Matrix Logic that we introduce the level of System into our consideration. We consider that Matrix Logic gives us the organizational level of specification of the autopoietic system. Stern talks about the autopoietic rings when his rings of matrix operators work on themselves.

When we introduce the anti-form it is by structurally reversing all the properties of the form. In terms of Laws of Form/Pattern it is the reversal of all the relations in a particular set of marks.

Form (⋁ ⋁) (⋀) ((⋁) ⋀ ⋀)
Anti-Form (⋁ ⋁ (⋀)) (⋁) (⋁ ⋁)

The specific elements may have different truth, identity, reality, and presence vectors associated with them:

R= r,r
I= i,i
T= t,t
P= p,p

The palaces may have 1, 0 or -1 (hidden) in either place in the vector. Since we want to be able to mark Being aspect vectors where ever we want in a series of marks we must introduce brackets to group things with the same vector designations. All this gives us the ability to introduce very complex equations that describe a system on the three levels of System, Form and Pattern. At the level of pattern we have Surreal Numbers. At the level of form we have both Laws of Form and Laws of Pattern differentiated by Braces. We have a quaternion of rotated marks. And we have the possibility of expressing forms and anti-forms. At the level of System we have the vector operators that allow us to express para-consistency and para-completeness. We introduce brackets to be able to label sets with the same aspect vectors. We get autopoietic organization at the level of the autopoietic system the way Stern says we do by the operation of matrices on each other in rings. At the level of pattern there are three different kinds of arrows that differentiate the kinds of patterns that the Surreal Numbers are representing.

This is a complex model. What can we say we archive with its different elements? First of all we have a model with specific formalisms that address what autopoietic unity is at each level. At the level of autopoietic form it is the 'mark' of G. Spencer Brown from his Laws of Form. At the level of Organization we have the Autopoiesis of rings of matrix operators that August Stern provides in Matrix Logic. It allows us to see how the features of the autopoietic system can be built up from the structural level of pre-entities by the coming together of the different kinds of patterning. It explains how content and form interact in terms of the containment of the surreal numbers by the marks. It explains how autopoietic form appears in the production of quaternion imaginary marks at the formal level. It explains how form and anti-form appear and how that is transformed into pattern and anti-pattern when we switch to the anti-rule set by reversing the laws of form to produced the laws of pattern. Here pattern means something different from the patternings at the structural level that are differentiated into kinds. Laws of Pattern refers to the emphasis of layering over multiplicity. Instead we might talk of the rule and the anti-rules that occur when we reverse the rules that apply to the algebra of the marks. We call this pattern because the emphasis on

layering rather than multiplicity focuses our attention on inwardness and immanence rather than externality and transcendence. This tends to make us look at patterns rather than forms. However the "Laws of Pattern" as an anti rule set is of a different nature than the patterning at the level of the pre-entities. We should not allow the use of the same word 'pattern' in two senses confuse us here.

If we talk in terms of rules and anti-rules then we clearly have Form and Anti-form under both regimes of rules. These forms are composed of marks and surrealistic arrows as contents. If we reverse the order of the marks and reverse the arrows we produce the anti form under a particular rule set. In laws of form there are two directions to our application of the rules. We can process them forward or backward depending if we want to elaborate the marks or simplify them. If we apply the marks to condense or elaborate it is possible to elaborate using rules and condense using anti-rules or vice versa. Forms and Anti-forms are assumed to cancel. But elaboration under rules and condensation under anti-rules may not lead to the same results. If we consider these elaborations and condensations of marks under the different rulesets then we can imagine that these are processes like the breathing of the breather soliton and anti-soliton pair operating on forms and anti-forms. We start with a form, we expand under one rule and then at some threshold we contract under the other anti-rules to get the complement form. If we do the same using the anti-form we the anti-complement form. The cycle from form to complement form and back to form is the dual of the cycle of the anti-form to the anti-complement form might be seen as the process of the anti-soliton falling into the soliton and vice versa. It produces a complex dynamic space in which forms and anti-forms cancel out but perhaps complements and anti-complements reinforce each other such that soliton and anti-soliton dynamics in the breather might be modeled. Laws of Form never addresses the question as to when expansion or condensation should be used. It merely identifies forms that are identical under a series of expansions and contraction operations. We have instead a notion that the autopoietic node is a super breather. The expansion and condensation is the breathing. That breathing contains within it two solitons that represent the two conjuncted dissipative systems which compose the autopoietic system. In this case the dissipation is occurring at the pattern level, so that one is a sign-process pair and the other is a value-structure pair. When these come together we get the production of the hyper cycle network of sign-value pairs on the other hand and on the other we get the process-structure pairs that define the autopoietic nodes. It is the activation of nodes by the hyper-cycles that produces the autopoietic system's operation. The autopoietic system is seeking homeostasis guided by the hyper-cycles. That homeostasis is a self-production which is ever different from itself, i.e. includes the genetic unfolding of the organism over a lifetime, just as the species unfolds in a different way over the generations. So the mechanism that allows self production is the core of the self-generating system that operates on the nodes as if they were what Goertzel calls Magician Systems. Such systems act as magicians in the sense that they all create each other in potential and then

their mutual potential creations cancel to give us the elements that exist in the next lifecycle. Self production is taken to be 'self' in the sense that Jung uses the term, as the totality of the conscious and the unconscious. The unconscious is seen as breaks in consciousness. Discontinuity is assumed and continuity is explained. Thus we are really modeling a meta-system not a continuous system.

When we look at the breathing then the sign-value soliton falls into the value-process soliton and vice versa producing an energy cycle that is very efficient. It is basically a self-producing trough where the trough for each soliton is the other soliton. This breather is static in spacetime instead of moving though an externally produced trough. It is an excellent picture of the cycle of self-production. When we try to model that using the laws of form-pattern and the patterning from the pre-entity level then we get a complex landscape where expansion and contraction, form and anti-form, rules and anti-rules intersect to produce a forms and complementarity forms or anti-forms and complementary anti-forms. This gives us something similar to the model of the Greimas square that we discussed earlier. In that model there is the form and the anti-form as contrast to the non-form. What we are saying here is that the non-form exists where the anti-rules apply instead of the rule. Thus the anti-non-form is the complement to a particular form that we get by expanding using the rule and contract using the anti-rule. This tells us that there is a chiasm between one possible anti-rule anti-form and another in this complex landscape of possible form/anti-form or rule/anti-rule transformations.

Rule:Form A	Anti-Rule:Form Non-Anti-A
Rule:Anti-Form Anti-A	Anti-Rule:Anti-Form Non-Anti-A

Because there are two sub-rules for each rule there are many combinations of these for expansion or contraction.

The relations between rules and anti-rules will be further treated in the Formalism Specification Appendix. The key point is that we are able to express all the levels that are covered by the theory of autopoiesis by means of these interconnected formalisms.

Autopoietic Organization = 4 aspects[3]
Autopoietic Forms = 4 forms[4]
Autopoietic Structure = 4 patternings[5]

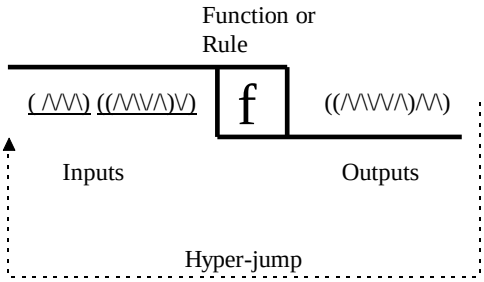
However, our analysis reveals that autopoietic theory does not actually reach the level of system without the *deus ex machina* of observers introduced from nowhere. In other words Autopoietic Theory as it stands is really about autopoietic forms and Structure and Organization are used to define the formal level but does not achieve the emergent level of the system except by the sophistry of the sudden introduction of observers from nowhere. Reflexive Autopoietic Theory is more natural in the sense that it merely defines system and meta-system and then reveals that at the point of balance between them exist

[3] Designation as Real, Present, Identical, True using Matrix Logic Vectors as superscripts on bracketed expressions. These aspects of Being have a quaternion relation to each other.
[4] The mark and the three imaginary marks associated with the ***i***, ***j***, ***k*** of the quaternion.
[5] The kinds of patterning that are associated with the aspects of Being are sign, value, process and structure.

the anomalous special systems. But we would still like to move up to the level of the system and understand that emergent level of phenomena in a natural way with a precise formalism. How that might be done will be addressed in the next section of this paper.

Autopoietic Systems

When we attempt to move up to the autopoietic ***systems*** ontological level we need to formulate what is necessary in terms of our understanding of Autopoietic Forms. These forms are represented by Spencer-Brown's marks. We need to find a way to understand the Forms of marks from a systematic perspective. We do this by extending again the concept of the mark to cover the difference between the mark in time and the mark in space. The mark in time can be thought of as a function while the mark in space may be thought of as the data that is operated on by the function.

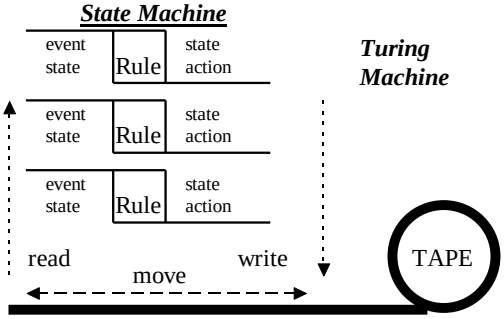


When we make this addition to the marks we are basically producing a functional programming language out of them. We are giving new meaning to their process dimension which is represented by the horizontal overhang. Now the overhang has been split so that the vertical stave can become a function and with its pre (left) and post (right) conditional input and output positions. We could just as well see this extension of the mark to include a dynamic as being described by rules instead of functions. The hyperjump that Spencer-Brown describes now becomes the means for iteration to occur and thus allows us to approximate a rudimentary programming language.

The function or rule marks may be combined in such a way to produce 'objects' by realizing that the marks as data are different from the marks as functions and by making the provision for segregating the data and operating on it via dedicated functions. Objects are combinations of forms in space (data) and forms in time (functions). Events can be seen as generated by the instances of the firings of the functions and Agents can be seen as the autonomy of rulesets which will allow simultaneous execution. A generalization of the Turing machine created by Gurevich called the Abstract State Machine Method allows us to describe arbitrarily complex systems by merely concatenating rules together. I have shown in a paper on this method that Rules have the special property of fusing all four methodological viewpoints into a single construct. A rule contains event, agent, function and data aspects in a single elegant construct. It also contains the different layers of information, i.e. data, information, knowledge, wisdom. Data is the values that are input and output by the rule. Information is the specific relations between data input and data output. Knowledge is the transformation that is made explicit in the rule. Experience comes from the side effects of the firing of the rule as Wisdom is the combination of knowledge and experience. Thus the rule is a very precise and singular conceptual element for building up networks of

causality. Our extension of the marks that functionalizes them sends us directly into the generalization of the Turing machine and allows us to produce images of arbitrarily complex systems composed by means of the Gurevich Abstract State Machine Method[6].

We can however restrict ourselves to the production of a traditional Turing machine then this would be effected by combining a Turing tape (infinite on one end and finite on the other) with a ruleset of functional marks. On the tape would be a series of marks and surreal content. The rules would operate on the marks on the tape reading and writing them to produce a simulation of a running program.



Once we have construed a way of transforming Laws of Form/Pattern marks into a dynamic simulation of a Turing machine then it is possible to define very precisely what a ***System*** is. The system is the showing and hiding of marks by the operation of the Turing machine program. What ever mark with its contents that the Turing machine is pointing at is the figure on the ground of the rest of the tape. A null mark is a blank spot on the tape. It is possible to go on then to define the meta-system as the Universal Turing Machine. In other words the meta-system is the means for switching between Turing machines, i.e. presentational systems that produce gestalts. The individual Turing machines are coded and written to tape, or if the universal Turing machine is a multiprocessor then it might allow different Turing machines to run simultaneously as different threads. A multi-processing Universal Turing Machine is equivalent to an operating system which is one of the best examples of a Meta-system as opposed to a System that acts like an application within an Operating System environment. In this way it is possible to see how we can step up quite naturally from the Laws of Form/Pattern level to the level of the system without introducing observers from nowhere.

Also this formulation allows us to consider the computational implications of the Hyper Kleinian Bottle. In a previous working paper the author has proposed that we might have Turing Machines with Mobius Tapes. Such a Turing Machine may simulate an Escher Waterfall and thus simulate a Dissipative System. With a mobius tape the concept of global non-duality and local duality is added to the concept of local coherence verses global incoherence. The tape takes the place of the trough of the soliton. The action of the Turing machine on the contents of the tape is the like the soliton wave. If the wave moves around the tape though the action of the Turing Machine state machine then it can come back to the same spot returning like the water in the waterfall over and over again. The difference between global incoherence and local coherence can be seen in the fact that the marching soliton waves around the

[6] http://www.eecs.umich.edu/gasm/

mobius strip will act like solitons or anti-solitons as they pass the same spot on either side of the strip. Duality and Non-duality comes from the fact that the strip is one-sided and the same in actuality even though it appears that it is different locally.

Now let us think about the equivalent of the Kleinian Bottle. We get this figure by conjuncting two Mobius Strips. What we lose is the distinction between the edges and it is inside/outside that becomes non-dual. In our picture of the solitons moving around the mobius Turing machine tape the solitons and anti-solitons would produce static breathers out of solitons that just happen to be opposite each other on the tapes. At this level we lose the distinction between the tape and the state machines that act on the tape. The tape is outside and the state machine is inside. If we lose the distinction between inside and outside, both becoming the same then data and program becomes the same. We will call the computer that does not know the difference between program and data the primal computer after a construct in Ben Goertzel's Web Mind[7] software.

The Primal Computer is an image of the Autopoietic System defined by the Quaternions, Breather Solitons and Kleinian Bottles. Such a computer does not know the difference between the inside (state machine) and outside (tape). It has come to embody global incoherence even though it may have nodes of coherence locally. Its inside/outside is non-dual globally while it is locally dual.

[7] cf Inteligenesis Company headed by Ben Goertzel See Wild Computing

Finally let us go up one more level to the Hyper-Kleinian Bottle. At that level there is the shared point of self-intersection between two Kleinian Bottles. This would be represented by two Kleinian Computers that did not know whether they were writing to their own program-data store or the other's program-data store. We will call this ambiguous situation Siamese Primal Computers. This is the level that Instantatons Super-Breathers arise. We can imagine them as the jumping back and forth between the two primal computers that do not know whether they are writing to their own or the Other's program-data area. The task switching between these two primal computers is like the jumping of the soliton though the potential trough in the field, i.e. the meta-system to another point in the system. In other words we can imagine that the solitons that are transferred are like agents that move between different networked computational environments. These agents are not just information but also programs that are transferred from one computational environment to another. Siamese Primal Computers are anathema to Software Engineering. They are instead the stuff of Artificial Intelligence that has been excluded from Software Engineering because self/other rewriting code is forbidden in normal software applications due to its indeterminate nature.

The self/other rewriting software system is a picture of the reflexive social level that are defined by the Quaternions, the Hyper-Kleinian Bottle and the Super-Breather. Such a computer is completely mixed up not knowing the difference between self and other within the milieu of the meta-system. In fact it is difficult

to tell such a computer from the computational environment because it is composed of agents that are continually transferring themselves form point to point in the environment spontaneously. It is as if the cells of a cellular automata were no longer fixed but could move in relation to each other freely while at the same time computing their neighborhood to give a value and also a vector of their own movement. Such a media would give extremely complex chaotic patterns. These patterns may be seen to be analogous to the octonion aambrot set that was hypothesized to exist by the author and first produced by Onar Aam. The Mandelbrot set is the most complex known mathematical object. It records the propensities in the field that connects the real and the imaginary numbers. These propensities appear as "lines of flight" which appear when a given point is iterated in this field. Similar structures exist for the quaternion and octonion. The quaterbrot and the Aambrot (or Octobrot) are multidimensional fractal structures. These define the propensities at these higher levels of complexity related to the Autopoietic and Reflexive special systems respectively.

The Artificial

This brings us to consider the nature of Artificial Intelligence, Artificial Life and Artificial Sociality in general. We have defined some very strange computational structures which we have called the primal computer and the Siamese primal computers. In general these kinds of structures are outlawed from Software Engineering. Software Engineering applies the minimal methods that arise between the four viewpoints on every real-time system (i.e. agent, function, event and data). These minimal methods and their application in Software Engineering has been explicated in Wild Software Metasystems by the author. The point that we would like to breach here is the fact that paradoxes appear in the field of minimal methods. These paradoxes become artificial intelligence techniques. It was confusing for some time why AI did not have methods like Software Engineering had. Then eventually I realized that it was because each AI technique was actually a paradox in the minimal methods layer and was excluded from Software Engineering for that reason. Examples of such paradoxes are:

- Self-rewriting Code
- Expert Systems
- Simulated Annealing
- Neural Nets
- Genetic Algorithms
- Etc.

Each of these is a programming technique that exhibits some opacity to the human intellect. To our intellect cognition is transparent. But the workings of these techniques is opaque in some sense. What is interesting is that we use these techniques as our examples of artificial intelligence. Thus artificial intelligences is naturally opaque to human cognition. This fact is extremely interesting because it means that when we begin developing artificially intelligent agents perhaps by combinations of these techniques we will not understand how they work nor how they reach the conclusions they reach. This opacity means that these agents will in fact be ***Alien*** to our own form of cognition. When we imagine that we will create artificially intelligent agents that will inhabit the realm of cyberspace then we are really talking about production of an alien culture of

agents in cyberspace, the first super multi-media medium, which will interact socially and will have opaque intelligences and will exhibit artificial life in that they have the illusion of being alive due to their motion thorough cyberspace and their transformative functioning.

Artificial Intelligence is opacity of programming techniques that are paradoxes in the software layer defined by the minimal methods between the four methodological viewpoints. Artificial Life is the independent autonomous functioning of these artificially intelligent agents. The fact that these agents are opaque to our cognition means that they are autopoieticly closed to us as observers. Thus Artificial Intelligence conjuncted with Artificial Life automatically yields Autopoietic Systems. When such systems interact socially then they automatically become reflexive and thus exist at the level of reflexive autopoietic systems. Thus Reflexive Autopoietic Systems Theory is the fundamental theory of interacting intelligent agents in the meta-system of cyberspace. Cyberspace is a world wide operating system for artificially intelligent social living agents. In defining that "region" the mobius Turing machine, primal computer (i.e. two conjuncted mobius Turing machines) and the Siamese primal computer pairs (i.e. four conjuncted mobius Turing machines) are archetypal. They occur in the meta-system or general economy of the global agent operating system that encompasses the earth through the electronic networks.

Emergent Meta-Systems Modeling

Once we realize that the Turing Machine is the means of defining the system and that the Universal Turing Machine is the means of defining the meta-system then we can begin to understand the problem of modeling the meta-system itself. For that purpose we think Systems Dynamics is the best candidate. Systems Dynamics gives a very accurate picture of the structure of the meta-system with its positive feedback loops that can go off in either a positive or negative direction which are balanced against the negative feedback loops that maintain homeostasis. Homeostasis is the nature of the autopoietic system. Positive feedback is the nature of the meta-system. Reflexive systems are in a meta-stable configuration between negative and positive feedback.

When we think about the meta-system and modeling it; it is necessary to understand that the systems dynamics equations are primarily applied to the dynamics of side-effects in the annihilation or cancellation process. The emergent meta-system is always canceling. But it does so in an environment where the associative and commutative properties have been lost. Because of that side-effects appear and those side effects form cascades. When the cascades of annihilations loop then we have what has been called by the author the anti-category of the annihilation mosaic. The annihilation mosaic answers the question how everything can be continually annihilating yet things seem to persist over time despite radical discontinuity. This is because side-effect cascades loop and these loops produce seeming persistence in the face of continual

relentless annihilation. Notice that this continual annihilation is the ever present fact of radical discontinuity which is a stronger precondition than entropy which is the fundamental assumption by Western Science. Special Systems Theory violates entropy by positing neg-entropy as a possibility. But that violation is based on a model of Emergent Meta-systems that can survive radical discontinuity though the looping of annihilation side-effect cascades forming annihilation mosaics.

Emergent meta-systems attempt to define the count of the swarm members left over after the annihilation process. This count might be seen to be determined using systems dynamics equations. In such equations the entire set of difference equations fires like an expert system in order to determine the count in the next EMS cycle of monads within the swarm. If we use Systems Dynamics equations to define these counts then we have a way to institute a model at the meta-system level without assuming continuity as normal deployment of systems dynamic modeling does. In this way we can model the general economy of the Emergent Meta-system and thus have a complete picture which gives our operating system or Universal Turing Machine a dynamic of its own. The Meta-system is not passive but has its own inner dynamic which warps the landscape of the environment significantly and constantly. The Agents afloat in this landscape must cope with the continual changes in the availability of resources governed by the systems dynamics equations that model the articulation of side-effects of cancellation.

Formalism Specification Appendix

In this section we will attempt to give a definition to our extension to the Laws of Form by G. Spencer-Brown.

Define Rule called *Laws of Form*

R1 ()()=()
R2 (())=ϕ

Define Anti-Rule called *Laws of Pattern*

A1 ()()=ϕ
A2 (())=()

The elements of the rules are:

- () - something
- (()) - layering
- ()() - multiplicity
- ϕ - null - absence or background

Rules can be applied to expand or contract the Form. The minimal expansion is:

Lemma 1
1) Start: ϕ
2) <A1: ()()
3) >A1: ϕ

Lemma 2
1) Start: ϕ
2) <R2: (())
3) >R2: ϕ

Lemma 1 is equivalent to virtual particles coming into and then popping out of existence. Lemma 2 is a layering popping into and out of existence.

We can construct a model of how something comes into existence as an alternation of the rules and anti-rules.

Lemma 3
1) Start: ϕ
2) <A1: ()()
3) >R1: ()
4) >A2: (())
5) >R2: ϕ

Lemma 4
1) Start: ϕ
2) <A2: (())
3) <R2: ()
4) >R1: ()()
5) >A1: ϕ

We notice that Lemma 3 and 4 are inverses of each other. They reverse time and all four rules get used. Between the forward time and backward time versions *something* is produced. Prior to the production of *something* there is either layering or multiplicity. After the production of *something* there is either layering or multiplicity. If there is multiplicity before then layering comes after or vice versa. In other words out of a layering comes *something* that produces a multiplicity before vanishing. Or out of nothing comes a multiplicity that produces *something* that produces a layering before vanishing. We can understand the prior and post states to *something* as the virtual particles prior to conservation. We can think of the layering as the wave state that is the opposite of the particles. If we think of it this way then we see that the (()) is the same as the ()(). It is merely the dual that we see in the wave-particle duality of matter that shows up in experiments. This means that the conserved something appears by going though a manifestation of the opposite virtual properties of matter in our formalism.

From these humble beginnings it is possible to see how we could build a complex formalism that allows the arbitrary expansion of these equations form the background and then their arbitrary collapse back into the background state of null [ϕ]. We can imagine that we might want to segregate the application of the rules from the anti-rules. In that case we would construct expressions using braces '{}' around the laws of pattern sections. Once we instituted variables then it would be possible to build up something like the algebra of the laws of form. However, our algebra would balance the immanence of the laws of pattern with the transcendence of laws of form. We can already see how these formalisms can operate independently or in concert. There must be meta-rules that define whether we stick to a single Rule set or mix them. Also there must be meta-rules that specify whether we expand or contract the expressions. The expressions arise naturally out of the background state and they then again naturally collapse back into the background. If we want to have a series of expansions and then a series of contractions then we need to specify in the meta-rules whether this is allowed or not. The cases we have cited so far move toward collapse as soon as possible. It is however possible to construct very elaborate expressions before we collapse back again into the background.

If we merely apply a single rule set then we hit a minimum expression that does not collapse back without retracing its steps. For instance:

Lemma 5
1) Start: ϕ
2) <A1: ()()
3) <A2: (())(()) >> expansion ((()))((()))
4) >A2: ()()
5) >A1: ϕ

Lemma 6

1) Start: ϕ
2) <R2: (())
3) <R1: (()()) >> expansion (()()()())
4) >R1: (())
5) >R2 ϕ

Notice that we can arbitrarily expand steps three in Lemma 5 & 6 but to return to the ground state we must reverse our steps unless we want to pass over to the opposite ruleset. The basic dynamics is that the Anti-rules produce infinitely deep nesting of two nodes while the Rules produce a multiplicity that grows in powers of two.

In Laws of form we encounter arbitrary series of symbols and have the possibility of an algebra if we introduce variables. But it is instructive to see what happens merely as we arise from the ground state itself. Without alternating between rule sets we must retrace our steps in the collapse from an arbitrary expansion. We might collapse back in a different pathway than our expansion and that is when we produce the theorems of the laws of form. Each such theorem has its laws of pattern dual. This is because laws of pattern is the dual category to laws of form. We will not work out those theorems here as they have already been worked out by G. Spencer Brown and by Varela and Kauffman in their ingenious form dynamics formulation of Laws of Form. What they did not realize was that there was a dual category that reverses the rules and that the super-category that includes both duals is far more interesting in its dynamics than the single Laws of Form algebra.

Instead of covering that old ground again of the development of the algebra of the combined dual categories we will continue to elaborate the formalism. We want to introduce content into the forms. We do that by introducing the concept of Surreal Numbers as the content of the forms. This is done by using the up and down arrows of surreal numbers. These could be seen as substitutes for Yin and Yang signs. By introducing these content diacritical marks we are distinguishing quantity from quality. Quantity participates in a N^2 matrix while quality is formed from the interpenetrations of the N elements giving a 2^N structure. We notice that the laws of form quantitative structure normally increases at a rate of 2^N. The layering laws of pattern structure increases linearly by just adding two layers each time. We introduce this qualitative structure by adding within the marks either up or down arrows.

(∨) or (/\)
(\/) ∨) or (∨ (∨))

Each layer can have its own qualitative marker. These markers may be as complex as necessary to express the quality. Normally though the qualitative layering should match the level of multiplicity. But it may be simplified to any heuristic level.

So for instance (/\ ) or (∨) is the first level with a single yin or yang marker This is adequate if there is only one form in multiplicity.

(∨ ∨) or (∨ /\ ) or (/\ ∨) or (/\ /\ )

This is the second heuristic level that is adequate if there are two forms in the multiplicity. We know that these two forms can just pop into existence: () ()

If we interpenetrate them then we get 2^N or 2^2 = 4 states. These four states

correspond to the for possible qualities that are represented by the combinations of up and down arrows.

So we might show the appearing forms with their qualities as they appear: (\/ /\ ) (/\ /\ ).

There is a transformational rule about heuristic qualities which is odd becomes even and even becomes odd as seen in Ilm al Raml (the science of the sands):

o + o = > o
oo + oo => o
o + oo => oo
oo + o => oo

or

/\ /\ => /\\/
\/\/ => \/\
/\\/ => \/\/
\/\ => /\ /\

Yang Major becomes Minor Yang
Yin Major becomes Minor Yin
Minor Yang becomes Major Yin
Minor Yin becomes Major Yang

This phenomena is called the rolling over of the opposites.

It is possible from the given pair of bigrams (\/ /\ ) (/\ /\ ) what the intersection of the qualities would be. We note that the first monad is odd while the second one is even. Odd and even give us even. Therefore the result is the flip of the even or (\/ \/).

(\/ \/) (\/ \/) => (/\ \/) => (/\ )
(\/ \/) (/\ /\ ) => (\/ /\ ) => (/\ )
(\/ \/) (/\ \/) => (/\ /\ ) => (\/)
(\/ \/) (\/ /\ ) => (\/ \/) => (\/)

(/\ /\ ) (\/ \/) => (\/ /\ )
(/\ /\ ) (/\ /\ ) => (/\ \/)
(/\ /\ ) (/\ \/) => (/\ /\ )
(/\ /\ ) (\/ /\ ) => (\/ /\ )

(\/ /\ ) (\/ \/) => (/\ /\ )
(\/ /\ ) (/\ /\ ) => (\/ \/)
(\/ /\ ) (/\ \/) => (\/ /\ )
(\/ /\ ) (\/ /\ ) => (/\ \/)

(/\ \/) (\/ \/) => (\/ \/)
(/\ \/) (/\ /\ ) => (/\ /\ )
(/\ \/) (/\ \/) => (/\ \/)
(/\ \/) (\/ /\ ) => (\/ /\ )

This same kind of patterning applies to the nesting. Except there we would expect it to reverse.

(/\ /\ (/\ /\ )) => (/\ \/) <right>
((/\ /\ ) /\ /\ ) => (\/ /\)

We can express non-associativeness by allowing the result to be inverted if the associated layering is on the right as opposed to the left. We can express non-commutativeness if we allow the result to flip when we cross back though the arrow.

(/\ \/) => (\/ \/ (/\ /\ ))
(\/ /\)) => ((/\ /\ ) \/ \/)

These complex qualitative dynamics gives a depth to the laws of form previously unavailable. This series of heruistics increases by steps as we interpenetrate more and more monads:

1) 2^1 Yin/Yang
2) 2^2 Major and Minor Yin\Yang
3) 2^3 Trigrams
4) 2^4 Ilm al Raml (Arab divination system)
5) 2^5 Yin/Yang by 5 hsing (transformations)
6) 2^6 I Ching (Chinese divination system)
7) 2^7 Bei (pacific island divination system)

These heuristic levels move to infinity down the Pascal Triangle and represent interpenetration of all things ultimately. But we can only handle about this level of complexity. Thus we simplify from all possible permutations of natural opposites to a level that sufficiently characterizes the level of independent objects we can hold in our short term

memory or less. These heuristic levels are the basis of Chinese and Islamic traditional sciences. They represent the levels of complexity of interpenetration of independent objects. We can only consider so much interpentration at once. In fact we can consider about as much as can be found in the reflexive system at the octonionic level which is at the threshold of seven imaginary things and one real thing. Short term memory is just big enough to deal with the reflexive milieu and no bigger.

Chinese traditional science is based on cycles taken from the platonic cycles of the third and fourth dimensions. We have to treat these platonic solids as lattices and then we see that they fit together such that each solid has some level of its lattice that connects to the lattice of another solid. The lattice of the penthedron of four dimensional space is crucial because that represents the five *hsing* and the ten celestial stems. It has the same group A5 as the icosahedron which has a cycle of 60. The icosahedron has the 12 that relates to the twelve branches. All these cycles used by Acupuncture that are found in the lattices of the Platonic Solids of the third and fourth dimensions fit into the greater cycle found in J2 the sporadic simple group that has 604800 members.

It is interesting that by combining the Surreal Numbers and treating them as diacritical marks that indicate qualitative states we can approximate the relation of form to content and that gives us some insight into the formalization of the heuristics of natural opposites. An excellent source for the definition of these dynamics is The Meaning of Man by Sidi Ali al-Jamal [Diwan Press]. Also one can refer to the Taoist classics especially the recently discovered ones. All of these dynamics may be approximated by our form-content formalism that connects Laws of Form/Pattern to Surreal Numbers.

We can think of the surreal landscape as being four dimensional. Its four dimensions are equivalent to the four kinds of patterns: sign, value, process, and structure. We can signify if a particular arrow is from one of these realms by placing a letter in the arrow: \p/, /p\, \s/, /s\, \v/, /v\, \t/, /t\ where 't' stands for 'structure'. This could also be handled by a different style of arrow for each kind of patterning. In this way we can have either pockets of a specific kind of pattern separated from another pocket: (\p/ /p\) (\s/ \s/). Or we may have mixed pockets of the kinds of pattern: ((/s\ /p\ \t/ /v\)). The mixed terms trace a path out into the four dimensional surreal surface space. That surreal space can be seen as the nature of Wild Being. As we have said previously in another context. Surreal numbers are composed of determinate, infinities, infintessimals and holes. If we multiply connect these infintesimals, or infinities, or holes we get a very wild landscape. But if we connect either infintesimals or infinities with holes then we get the rudimentary dissipative structures because series of numbers come out of nowhere. From this we can build up autopoietic and reflexive structures out of these dissipations. Therefore surreal space which is an image of Wild Being gives us directly a model from which we can derive the special systems. The forms that occur with Laws of Form (rules) or Laws of Pattern (anti-rules) merely give us the means to mix this spacetime partitioning it and manipulating it. This manipulation has a

Pure Presence aspect given where we draw our distinctions in the Wild Being substrate of surreal spacetime. The Process Being aspect appears in how far it is between opening and closing parentheses. The Hyper Aspect has to do with the jumping into a certain point in an expression from another point. It is the hyper jumps that cause the mixing. We can represent this jumping by having 'from' and 'to' hyperlinks in the expression: (((f1)((t2))((()(f2)) t1)). Or if we use marks Spencer-Brown suggests lines underneath the expression that has an arrow. If we are to jump then we must be jumping though one of the other dimensions so the hyperlinks justify the interpretation of four dimensional surreal space. If the Surreal Space is four dimensional then it is clear that we also need four dimensional marks. We have already suggested the notation of the rotated marks for this. But if we are expressing the marks in a row using parentheses then the notation becomes cumbersome. In stead of this we can use four expressions one for each axis in the four dimensional space. The marks represent the segmentation of that axis.

S: (())(()).(())()()(()) for the sign dimension
P: ()()(())().(())((())) for the process dimension
V: ()()()().()()((())) for the value dimension
T: (())()().((())) for the structure dimension

A period represents the zero point on each axis. Four expressions with periods would completely segment the four dimensional Surreal space.

We will also allow Forms and Anti-forms cancel each other. When we have a form ((/\/\)\/) then we will assume that the anti-form is (/\(\/\/)). When ever duals like this occur they cancel each other leaving the void. For these purposes sign is the opposite of value and process is the opposite of structure if the arrows are marked. So (\s//p\) cancels (\t//v\).

The next step is to add on Matrix Logic. This is done by giving superscripts to the expressions and placing in brackets those that have the same aspect designation: $[(\s/)]^{R0,-1}$ the possible superscripts are R for reality, T for truth, I for identity, and P for presence. The vector positions can take on the values of 1 for asserted, 0 for denied, and -1 for hidden. Matrix Logic has a complete logic for manifestation. It takes the positions of the game of what hand holds the coin. There are the following positions:

Right hand full and out = x,1
Right hand empty and out = x,0
Left hand full and out = 1, x
Left hand empty and out = 0,x
Right hand empty and Left hand full both out = 1,0
Right hand full and Left hand empty both out = 0,1
Both hands full and out = 1,1
Both hands empty and out = 0.0
Both hands hidden = -1,-1
Right hand hidden Left hand Full and out = 1,-1
Right hand hidden Left hand Empty and out = 0,-1
Left hand hidden Right hand Full and out -1, 1
Left hand hidden Right hand Empty and out -1,0

Matrix Logic uses truth matrices on aspect vectors to manipulate them. Vectors may take the Bra or Ket orientation. Matrix operations may operate on other matrices as well as the vectors. A matrix operation on two vectors may yield either a aspect scalar or a matrix. Reduction to a scalar or expansion to a matrix are seen as duals in the meta-system.

A given expression may have four associated superscripts $[()()(())]^{R0,1,\ T0,-1,\ P1,1,\ I0,0.}$ The aspects form another four dimensional space where the different dimensions are manifestation lattices. Matrices may be applied to mixed sets of aspects. This gives the organizational

power of meta-logic to the Autopoietic System. When something is asserted and denied at the same time this is para-consistency. When something neither asserted or denied at the same time this is para-completeness. When assertions and denials are both hidden then we have the thing existing in potentia. These may be true of any of the aspects of Being.

Since the kinds of pattern arise as pre-entities from the aspects of Being we have a closed circle here. Pre-entities take the form of sign, value, process, structure and these appear in a four dimensional surreal space. This space is a rhizome of multiple connections. The space is segmented by marks. The marks must be four dimensional to handle the segmentation of the space. But their four dimensionality is quaternionic because they have to produce autopoietic unity at the level of form. But these forms participation in manifestation and manifestation is segmented by the aspects. Thus we get four different aspect designations that are manipulated by the logical matrices. Autopoietic organization comes from the rings of operators Stern describes. Because the aspects of Being are themselves quaternionic that allows there to be autopoietic unity at the system level. Reality, Identity and Truth are the quaternionic imaginaries within Being. So the different aspects again gain unity within manifestation at the system level, not just the level of the autopoietic form.

If we introduce observers to the level of the autopoietic system it would use either the Observer Mechanics or Jumaries Information, Subject, System or some mixture of both. Both of these formalisms relativize observers. Jumarie is the preferred formalism for observers.

All this has described a formalism that allows us to describe the Autopoietic System in detail starting from the patterning level (structure) and moving though autopoietic form to end up at the system level (organization). It is complex and not worked out in detail. But this sketch allows the causal observer to see that it is possible that by interlocking these formalisms we can construct a model of the autopoietic system that goes well beyond what Spencer Brown proposed and Varela and Kauffman elaborated.

Organization describes the manifestation of the forms with their contents within Being. Being includes the four aspects (reality, identity, truth and presence). These define the relationship between system and meta-system in the sense that the meta-system is a filter on the systems that it encompasses. The meta-system is continually testing the reality of its systems while the systems are continually attempting to establish their truth. The systems are concerned with their own identity while the meta-system is concerned with the presencing of the system within the meta-system. Thus the meta-system is concerned primarily with physus while the system is concerned primarily with logos. The four aspects of reality are their means of mutual appropriation within the overall structure of Being which has four aspects and four kinds. As we go up the meta-levels of kinds of Being we encounter at each level a different interpretation of each aspect of Being. Reality is different and has emergent aspects at each meta-level of Being. The same is true of all the aspects of Being.

As was mentioned in the main body of the paper, in order move to the level of system we need a different model from the generalized model of manifestation that we achieve by employing Matrix Logic with regard to each aspect of Being. Organization is the ordering within manifestation of the forms with their associated contents. But to move to the level of system we need to develop the idea of an object out of the lower level concept of the 'mark'. We have achieved this by making each mark a rule or function which as part of its processing in time produces a result. Then we can employ the hyper-jumps that G. Spencer-Brown suggests as the means of creating recursion and though that all the other programming constructs. We then concatenate these rule-marks to produce a Turing Machine as previously described. That leads us into the series of computational unfoldings that mirrors the hierarchy of the special systems on up to the meta-system level which is modeled by the universal Turing Machine.